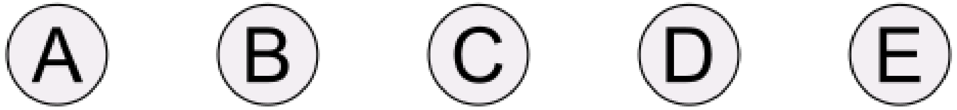
fair allocation of indivisible Chores

Rohit Vaish

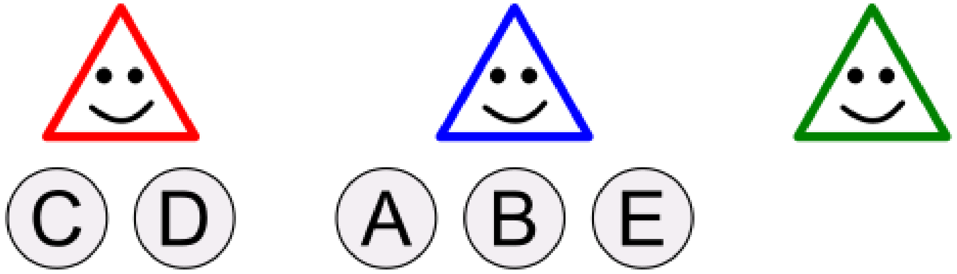
The Model

Set of agents

Set of indivisible items



Allocation



Valuation Function

For three items A B C

{ } = 0

{A} = 1

{B} = 0

{C} = 2

{A B} = 1

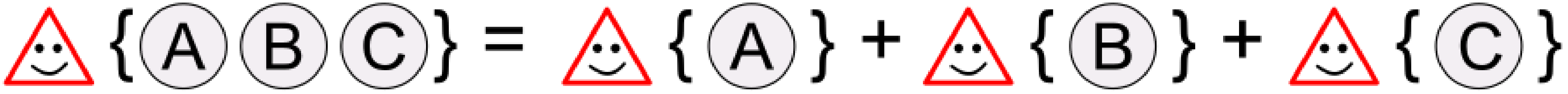
{B C} = 2

{A C} = 3

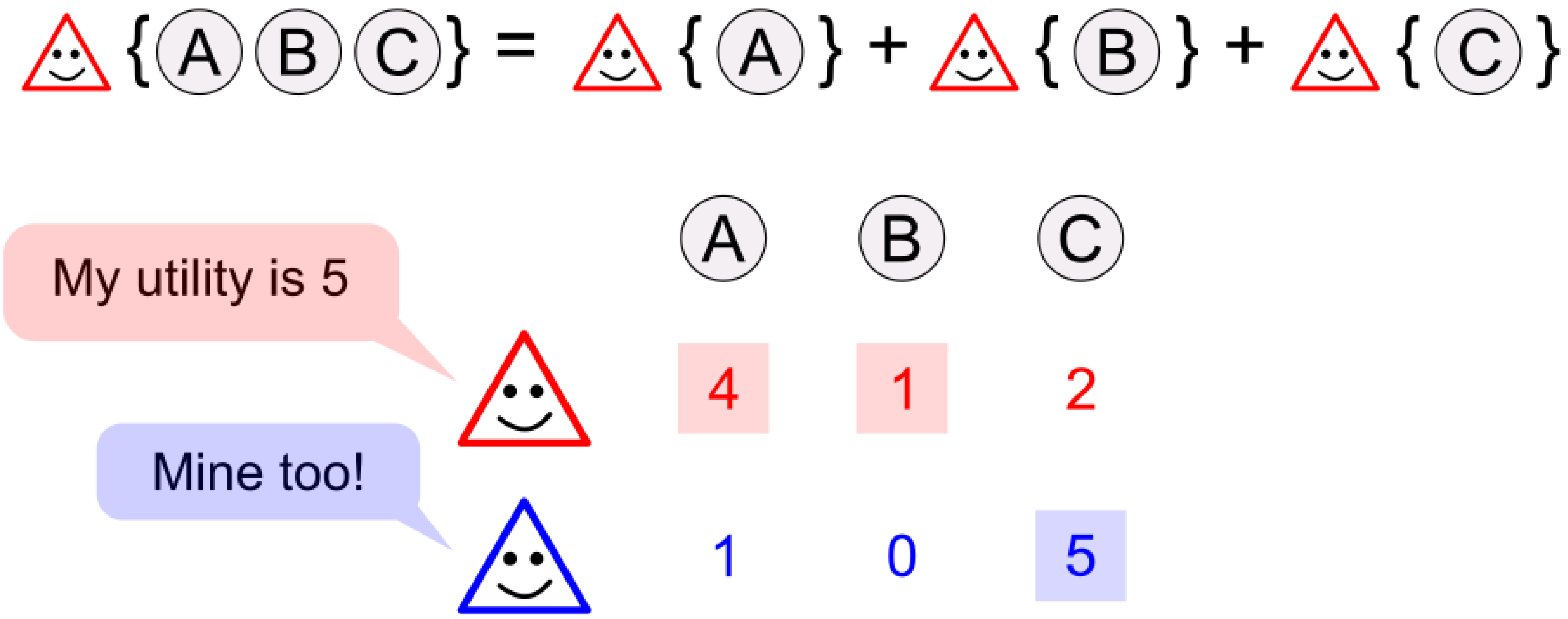
{A B C} = 3

Description grows *exponentially* with the number of items!

Additive Valuations



Additive Valuations



Description grows *linearly* with the number of items.

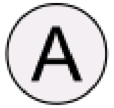
Marginal Value

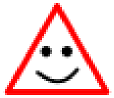
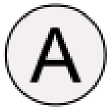
Marginal value of Ⓐ for △ with respect to { Ⓑ Ⓒ }

△ Ⓐ | { Ⓑ Ⓒ } = △ { Ⓐ Ⓑ Ⓒ } - △ { Ⓑ Ⓒ }

Types of Resources

Types of Resources

The item  is a good for  if for *all* subsets of items S

  $\mid S \geq 0$

Types of Resources

The item A is a good for △ if for *all* subsets of items S

$$\triangle\ A \mid S \geq 0$$

E.g., an extra GB of cloud storage

2GB 5GB 15GB 10GB

Types of Resources

The item Ⓑ is a chore for △ if for *all* subsets of items S

$$\triangle\ \text{Ⓑ} \mid S \leq 0$$

E.g., a dish that you forgot to wash

Types of Resources

Good for one agent, chore for another: Mixed items

E.g., service charge in restaurant bills

Types of Resources

If all items are goods for all agents: Goods instance

If all items are chores for all agents: Chores instance

Otherwise: Mixed instance

Types of Resources

Goods

Chores

Mixed

Types of Valuation Functions

Goods

Chores

Mixed

Types of Valuation Functions

Goods

=

Monotone ↑

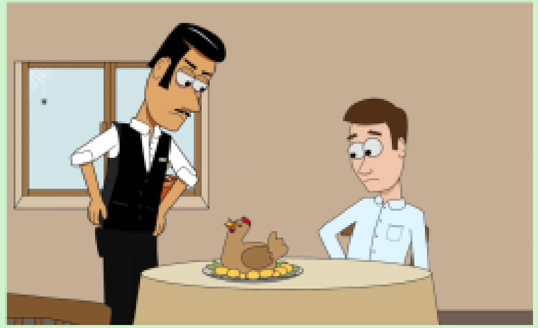
 $S \geq$  T

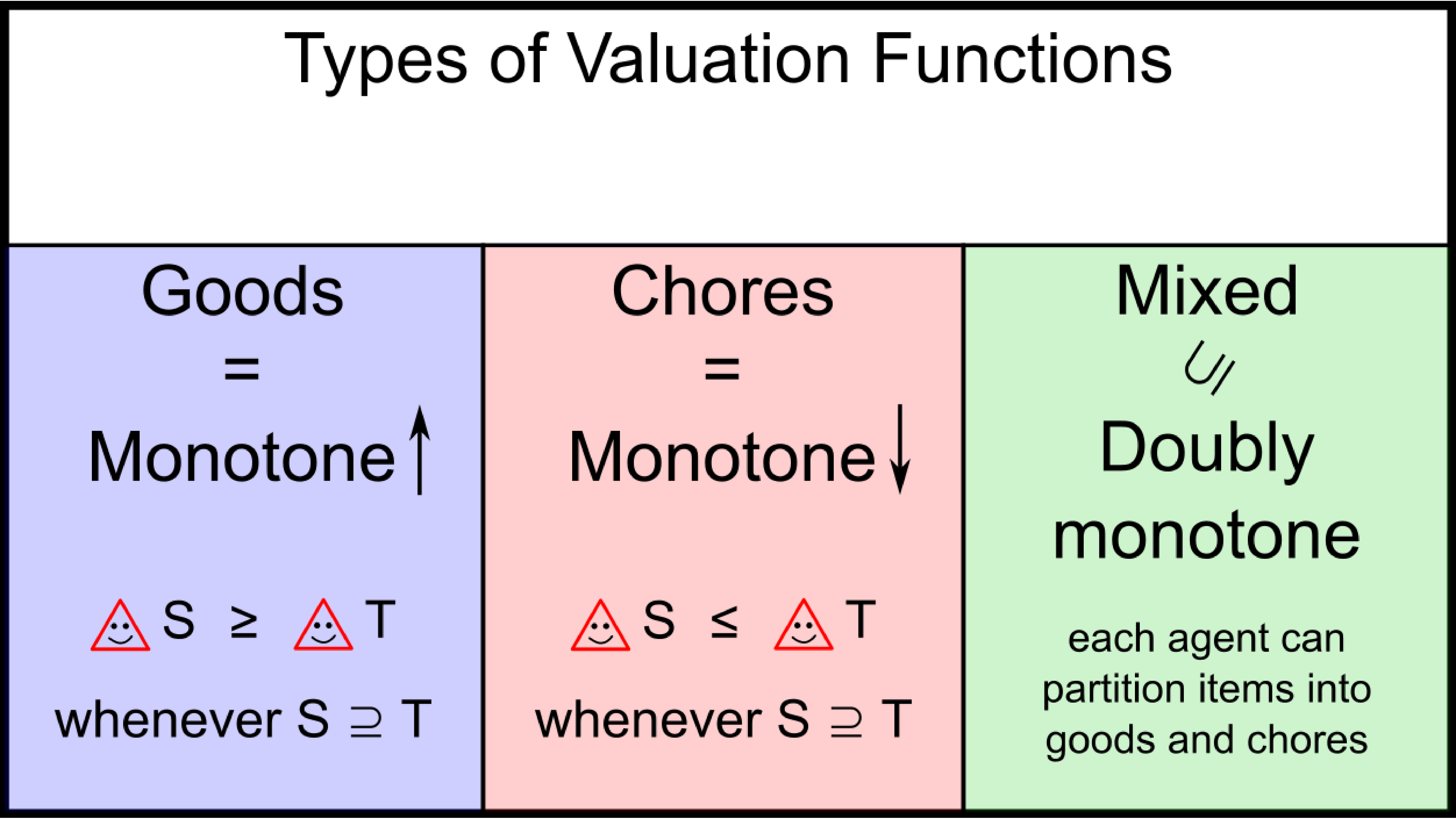
whenever $S \supseteq T$

Chores

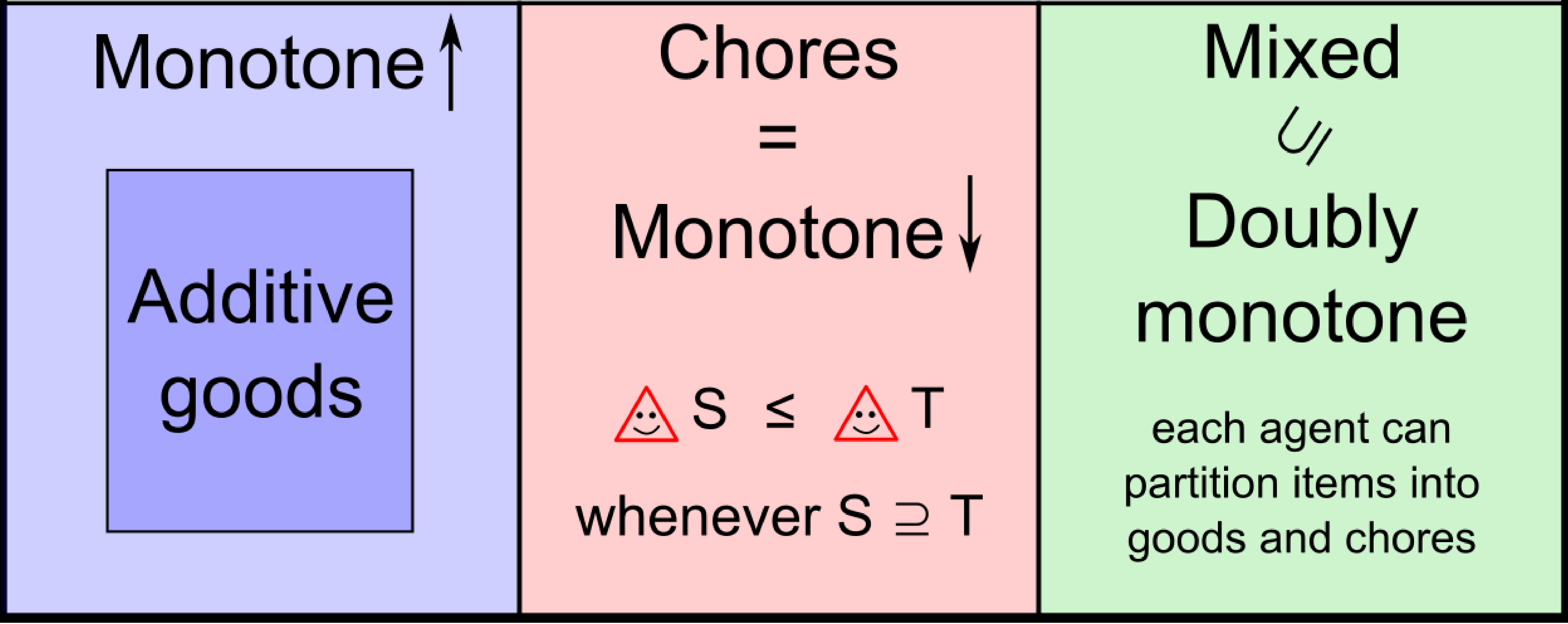
Mixed

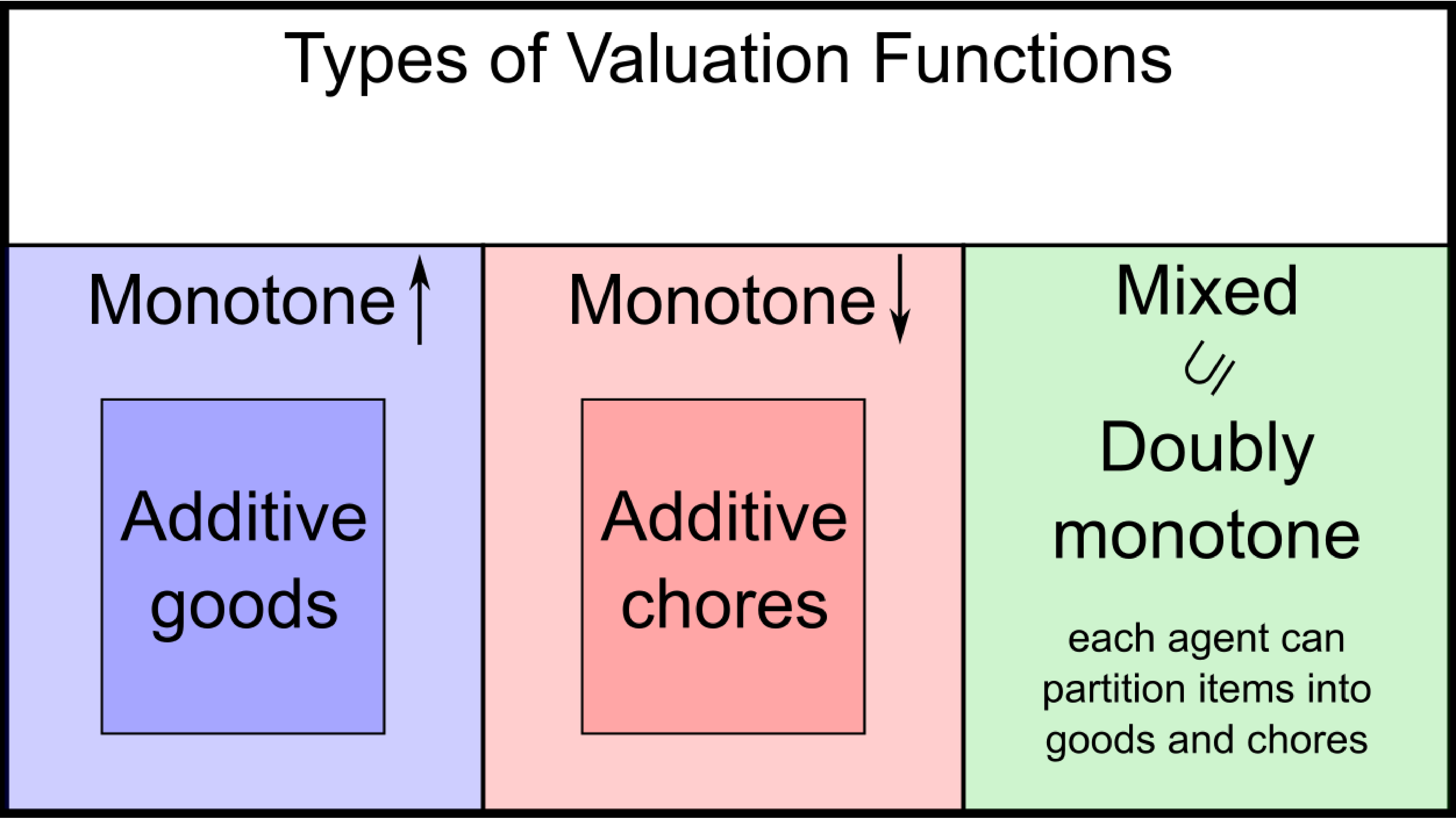
Types of Valuation Functions

Goods	Chores	Mixed
=	=	
Monotone ↑	Monotone ↓	
$\triangle S \geq \triangle T$	$\triangle S \leq \triangle T$	
whenever $S \supseteq T$	whenever $S \supseteq T$	


Types of Valuation Functions
Goods
=
Monotone↑
S ≥ T
whenever S ⊇ T
Chores
=
Monotone↓
S ≤ T
whenever S ⊇ T
Mixed
⊆
Doubly monotone
each agent can partition items into goods and chores

Types of Valuation Functions

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Monotone↑
Additive goods
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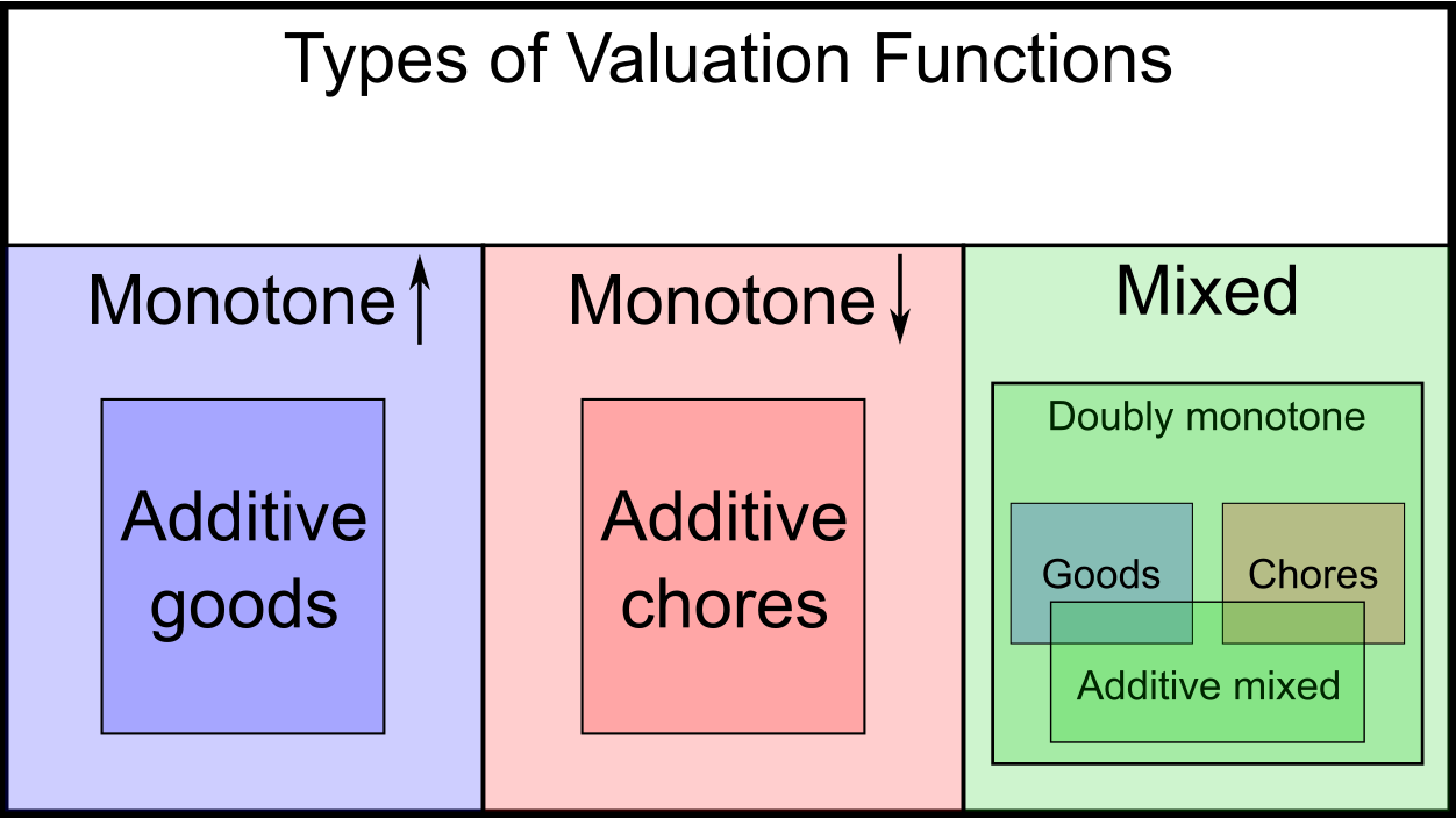
Types of Valuation Functions
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Monotone↓
Additive chores
Mixed
Doubly monotone
Goods
Chores
Additive mixed

Types of Valuation Functions

Under additive valuations

Goods

	A	B	C
	4	1	2
	1	0	5

Chores

	A	B	C
	-1	0	-2
	-5	-1	-1

Mixed

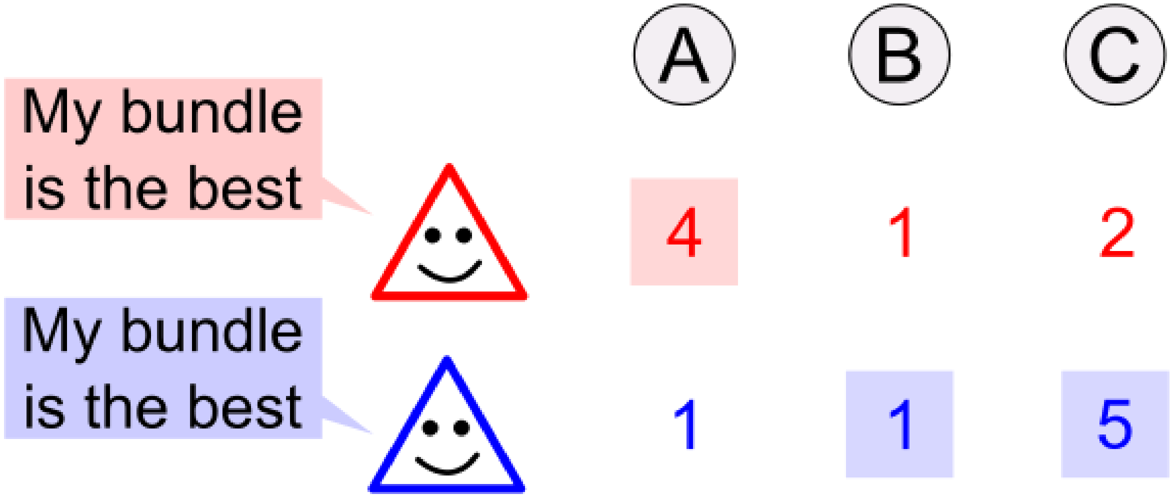
	A	B	C
	1	1	-1
	-2	0	-2

Fairness Notions

Envy-Freeness

[Gamow and Stern, 1958; Foley, 1967]

Each agent prefers its own bundle over that of any other agent.



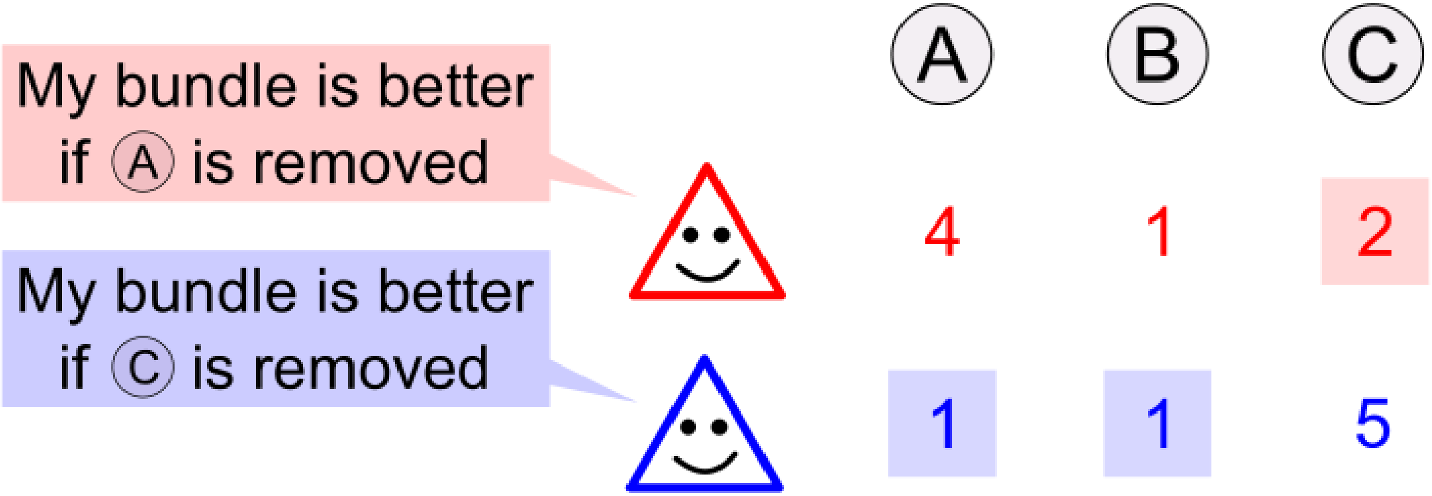
Not guaranteed to exist (two agents, one good)

Checking whether an EF allocation exists is NP-complete

Envy-Freeness Up To One Good

[Budish, 2011]

Envy can be eliminated by removing some good in the envied bundle.

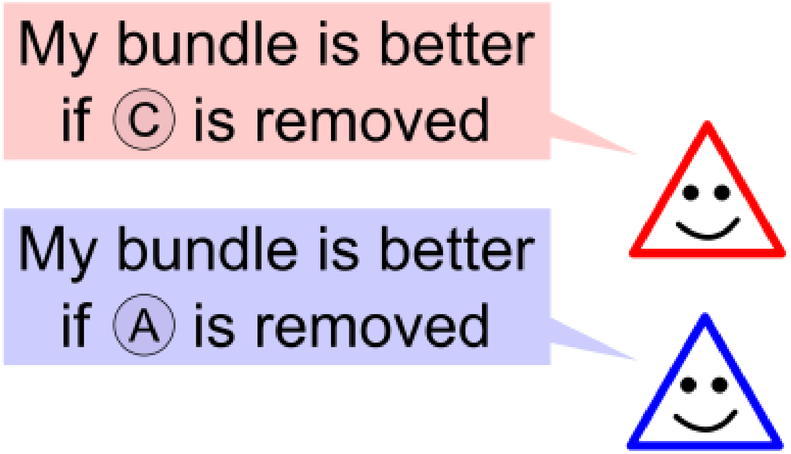


Guaranteed to exist and efficiently computable

Envy-Freeness Up To One Chore

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

Eliminate envy by removing some chore in the envious agent's bundle.



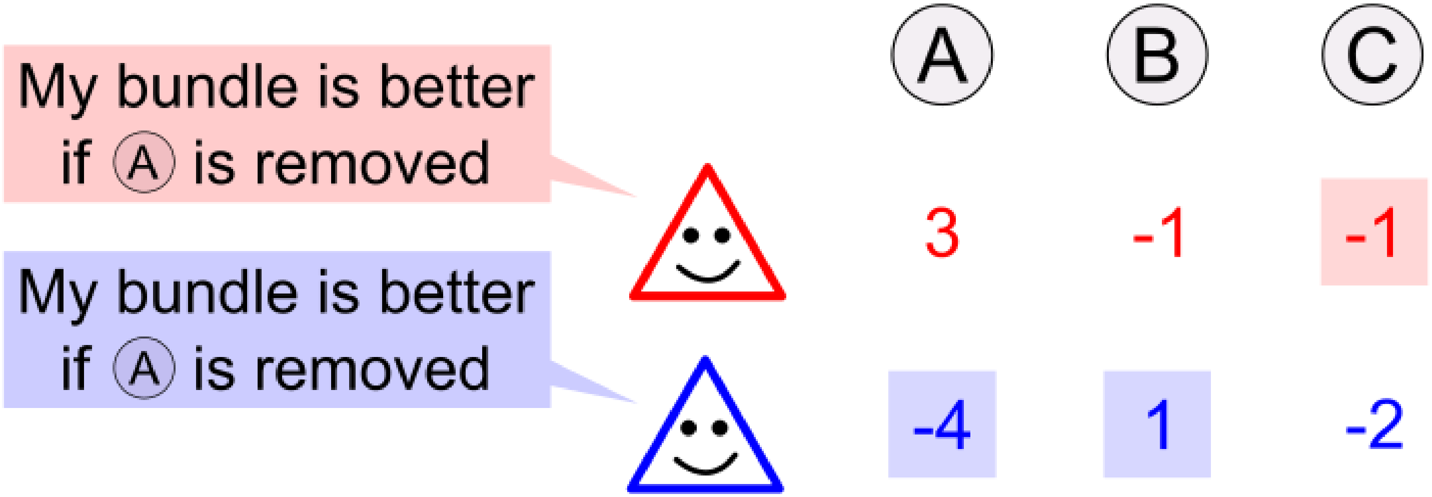
	A	B	C
	-1	-1	-3
	-4	-1	-2

Allocation $A = (A_1, \ldots, A_n)$ is EF1 if for every pair of agents i, k, there exists a chore $j \in A_i$ such that $v_i(A_i \setminus \{j\}) \geq v_i(A_k)$.

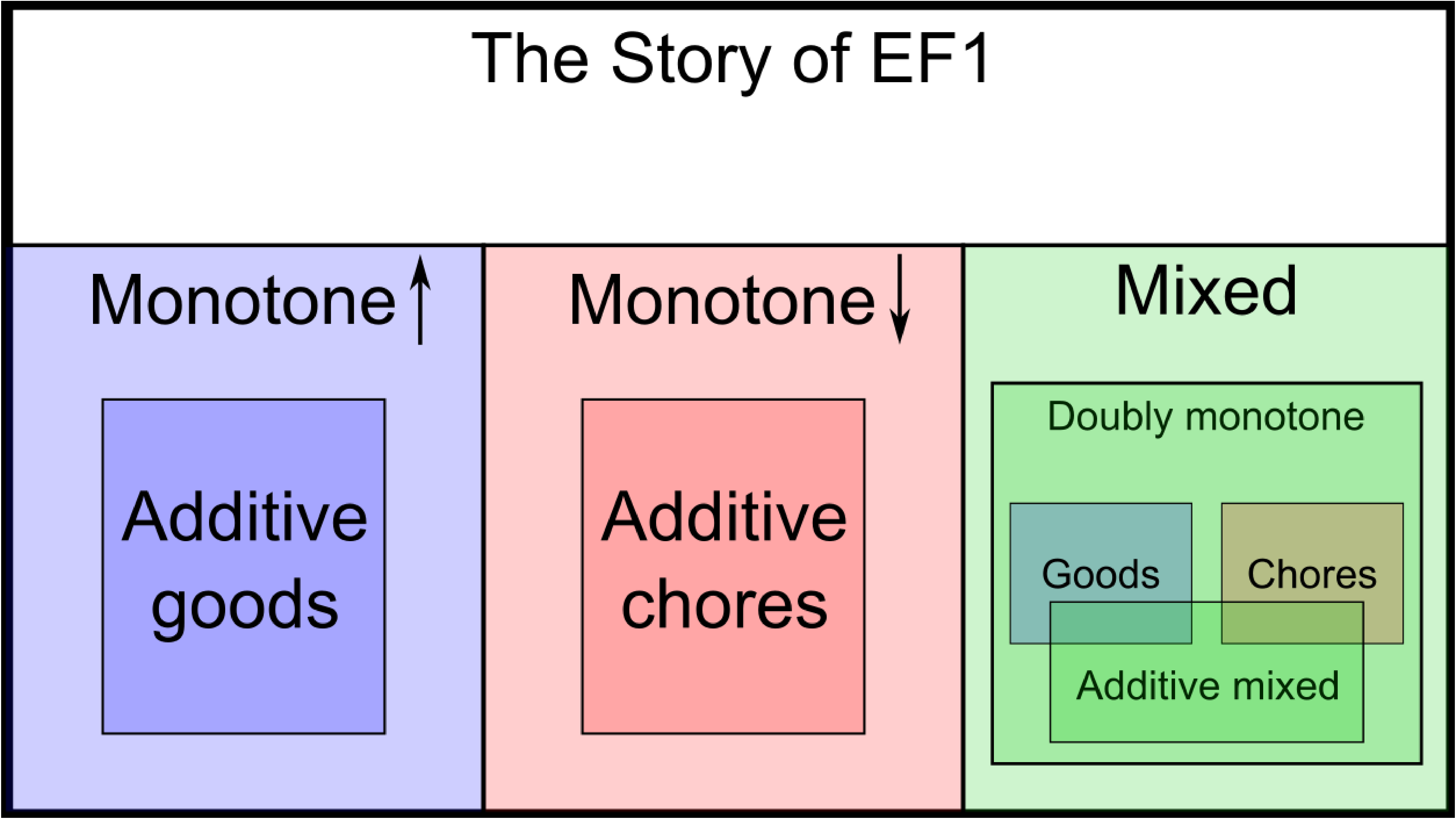
Envy-Freeness Up To One Item

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

Eliminate envy by removing some "good" in the envied bundle or some "chore" in the envious agent's bundle.



Allocation $A = (A_1, \ldots, A_n)$ is EF1 if for every pair of agents i, k, there exists an item $j \in A_i \cup A_k$ s.t. $v_i(A_i \setminus \{j\}) \geq v_i(A_k \setminus \{j\})$.


The Story of EF1
Monotone↑
Additive goods
Monotone↓
Additive chores
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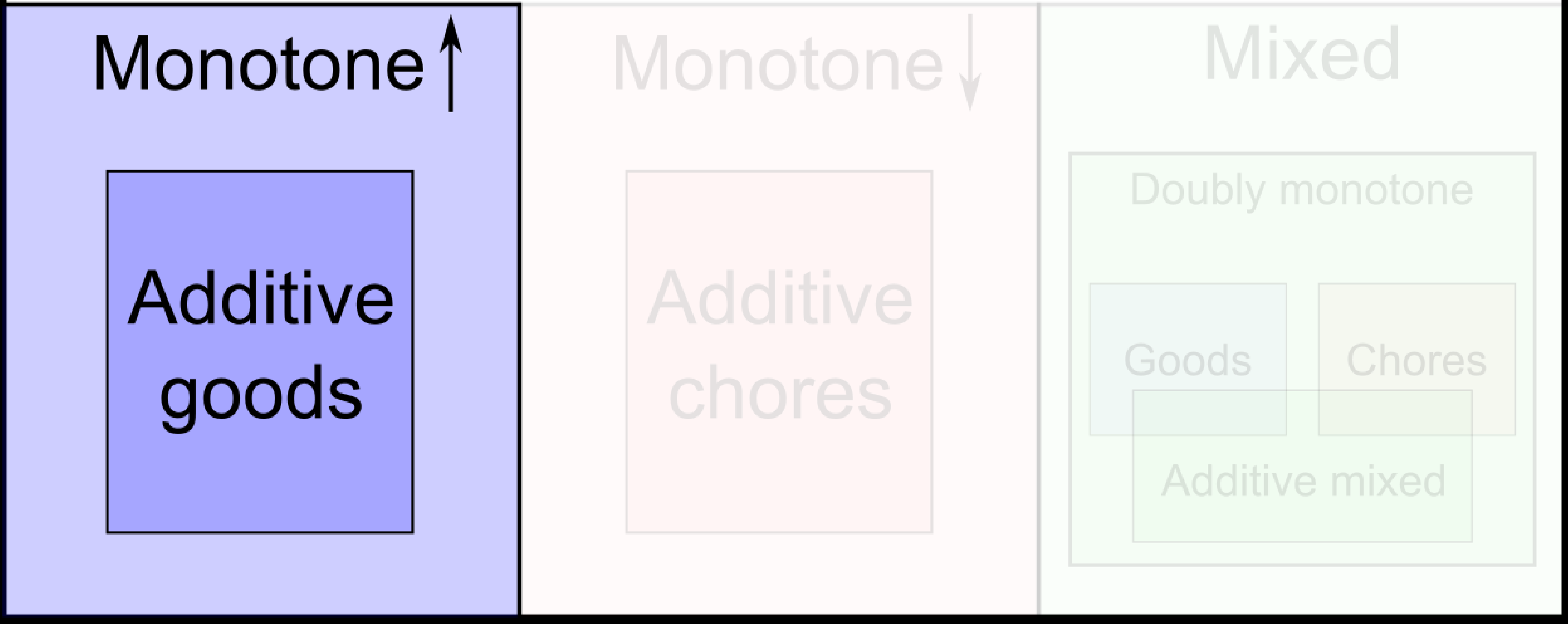
Doubly monotone
Goods
Chores
Additive mixed

The Story of EF1



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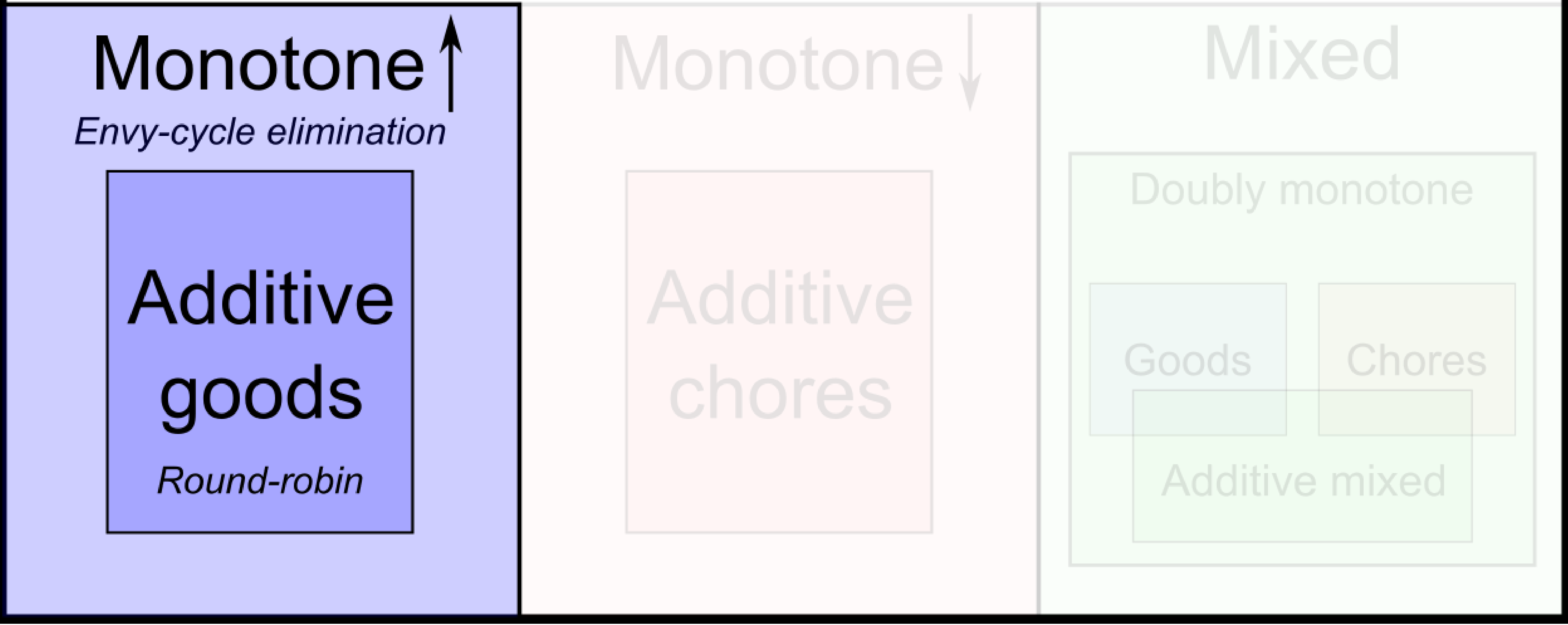

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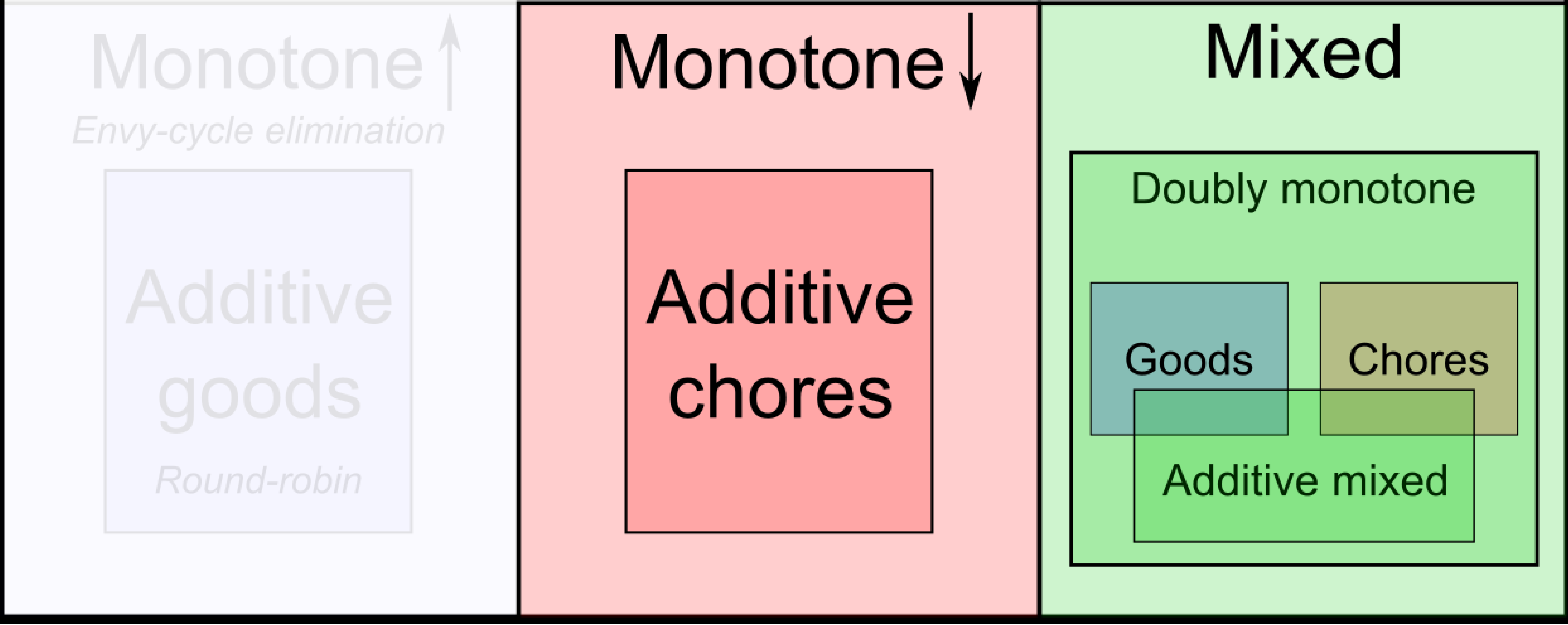

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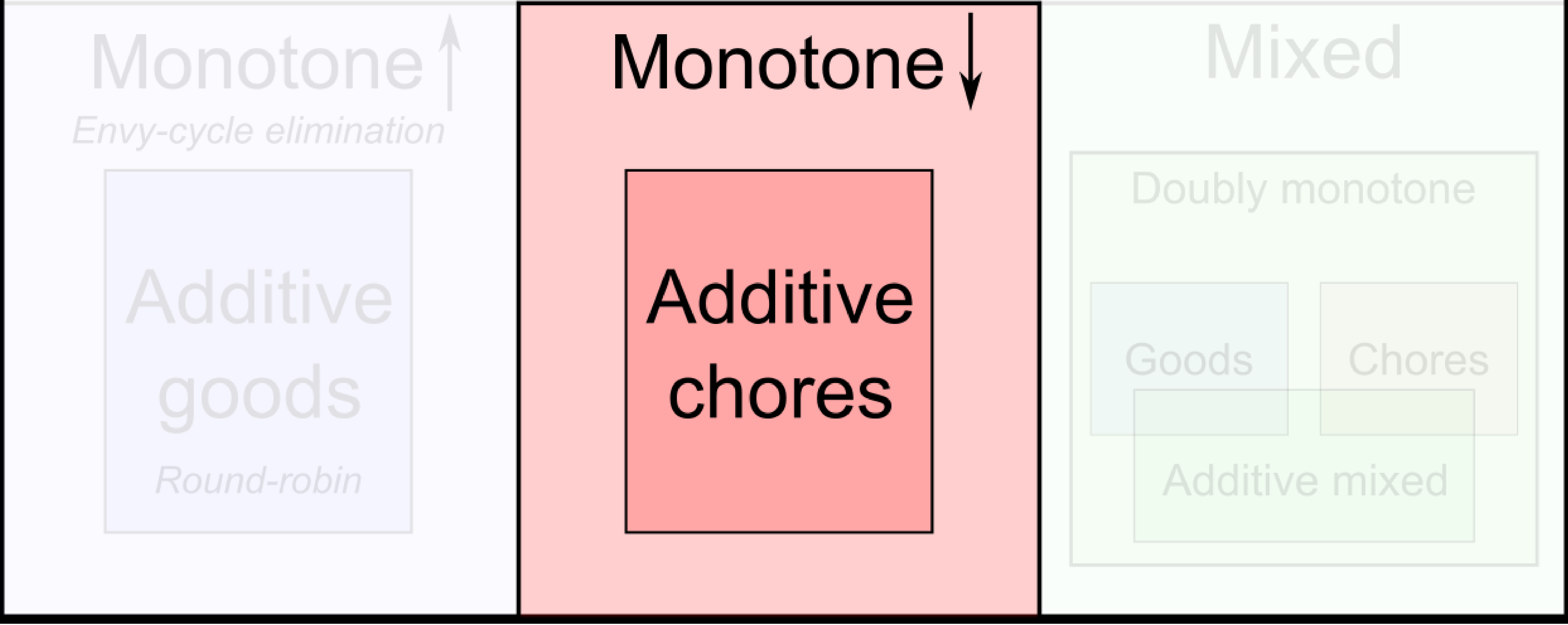
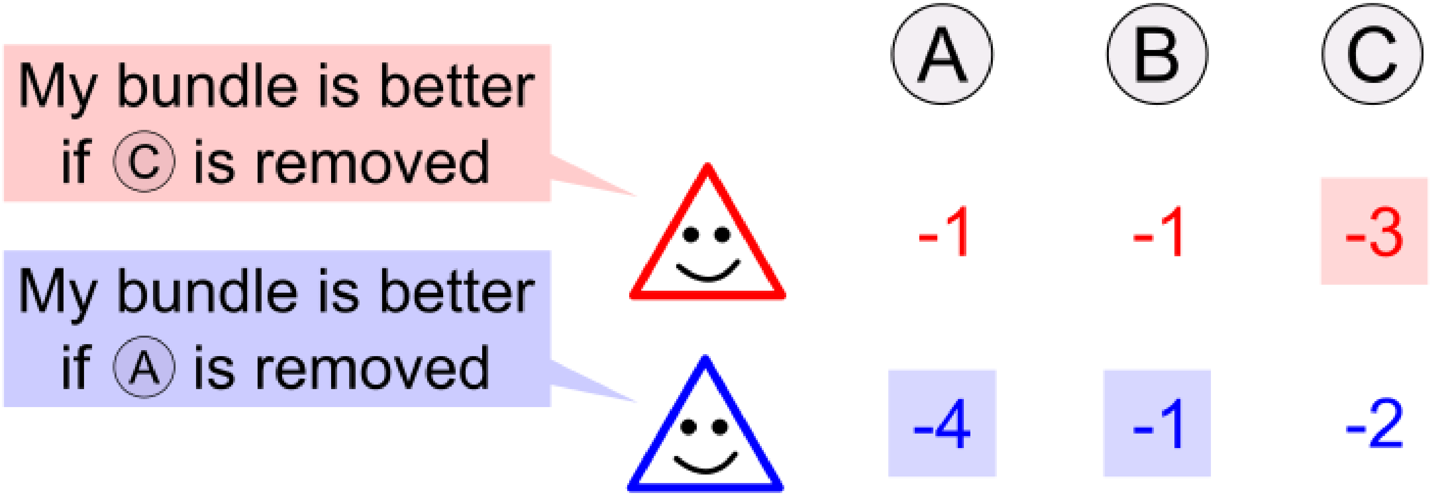


The Story of EF1

Envy-Freeness Up To One Chore

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

Eliminate envy by removing some chore in the envious agent's bundle.



Allocation $A = (A_1, \ldots, A_n)$ is EF1 if for every pair of agents i, k, there exists a chore $j \in A_i$ such that $v_i(A_i \setminus \{j\}) \geq v_i(A_k)$.

For additive chores, the allocation computed by round-robin algorithm satisfies EF1.

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	A	B	C	D	E
	-4	-1	-3	-2	-4
	0	-1	-5	-2	-1
	-4	-2	-5	-3	-1

For additive chores, the allocation computed by round-robin algorithm satisfies EF1.

	A	B	C	D	E
	-4	-1	-3	-2	-4
	0	-1	-5	-2	-1
	-4	-2	-5	-3	-1

For additive chores, the allocation computed by round-robin algorithm satisfies EF1.

	A	B	C	D	E
	-4	-1	-3	-2	-4
	0	-1	-5	-2	-1
	-4	-2	-5	-3	-1

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	A	B	C	D	E
	-4	-1	-3	-2	-4
	0	-1	-5	-2	-1
	-4	-2	-5	-3	-1

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Fix a pair of agents (r,b). Analyze envy of r towards b.

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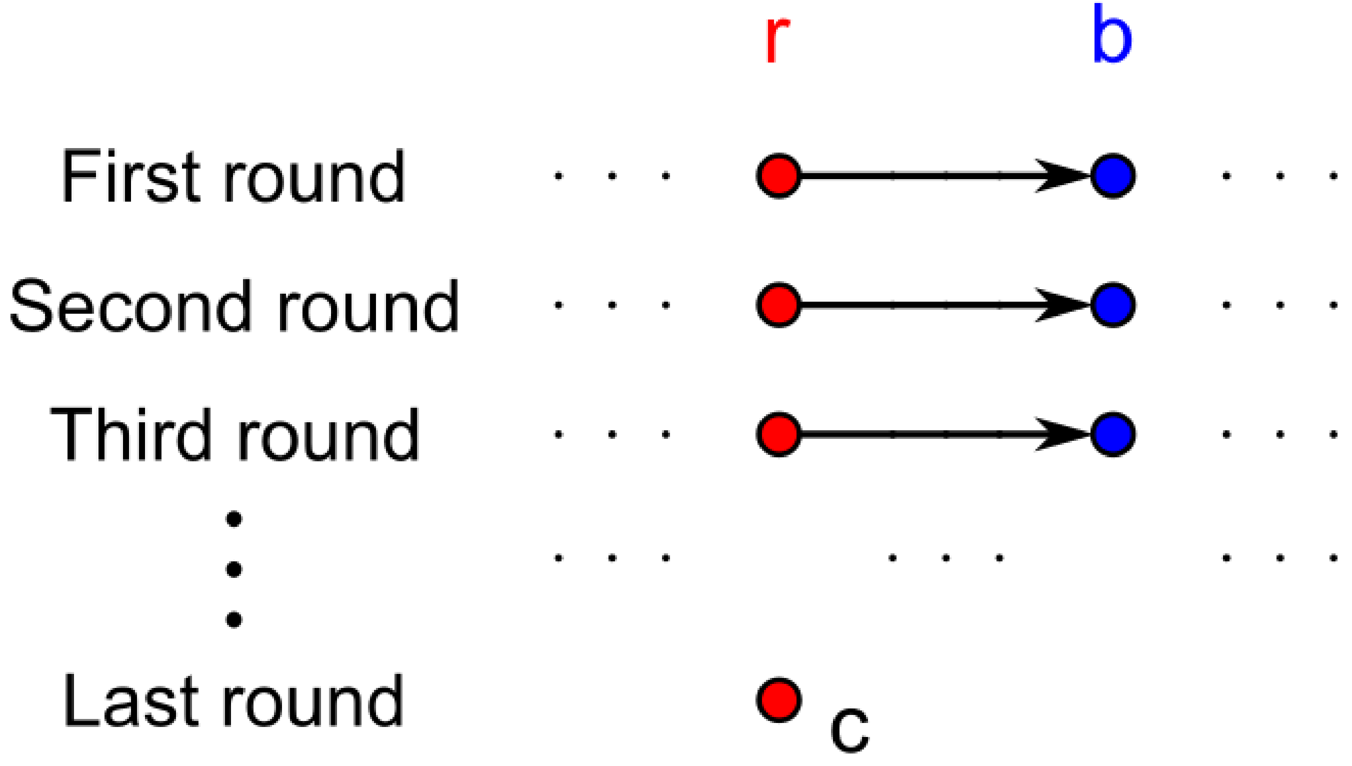

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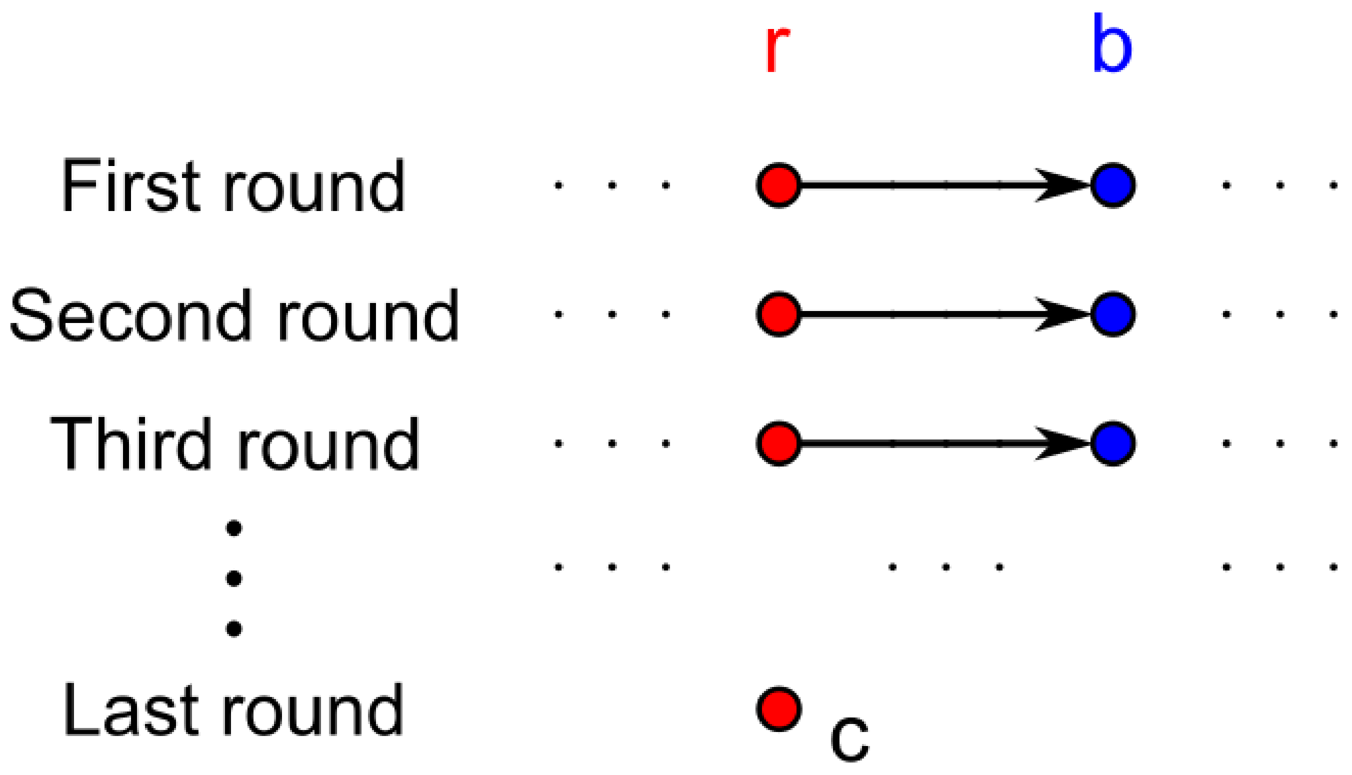

If r precedes b: Then, by additivity, $v_r(A_r \setminus \{c\}) \geq v_r(A_b)$.

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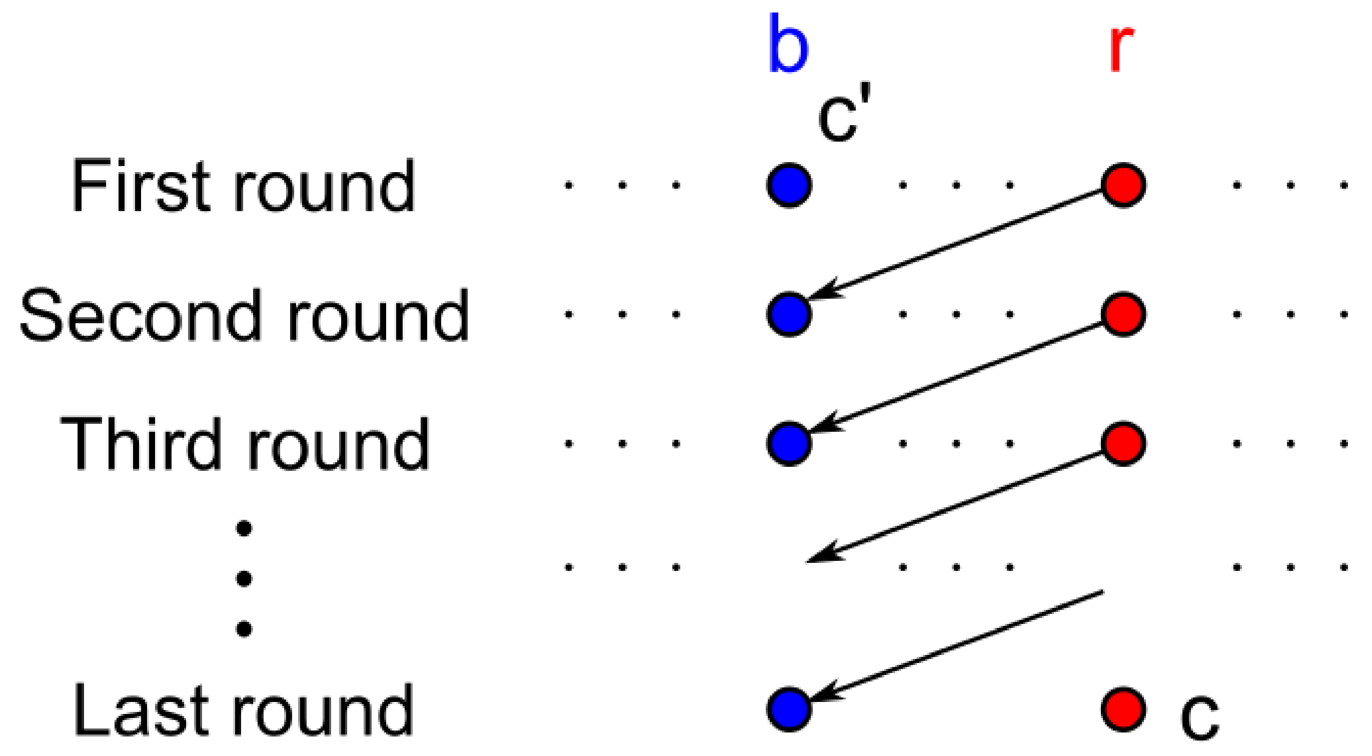

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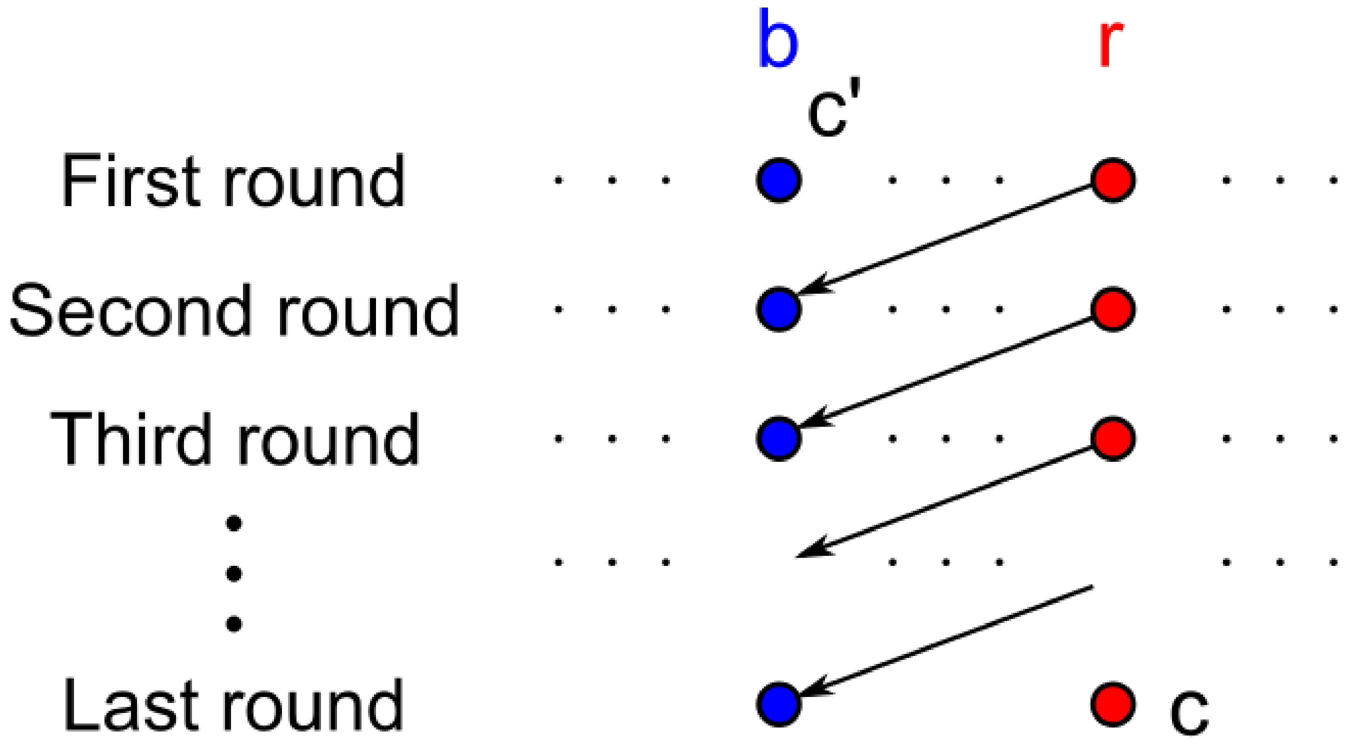

If b precedes r: Again, by additivity, $v_r(A_r\setminus\{c\}) \geq v_r(A_b\setminus\{c'\}) \geq v_r(A_b)$.

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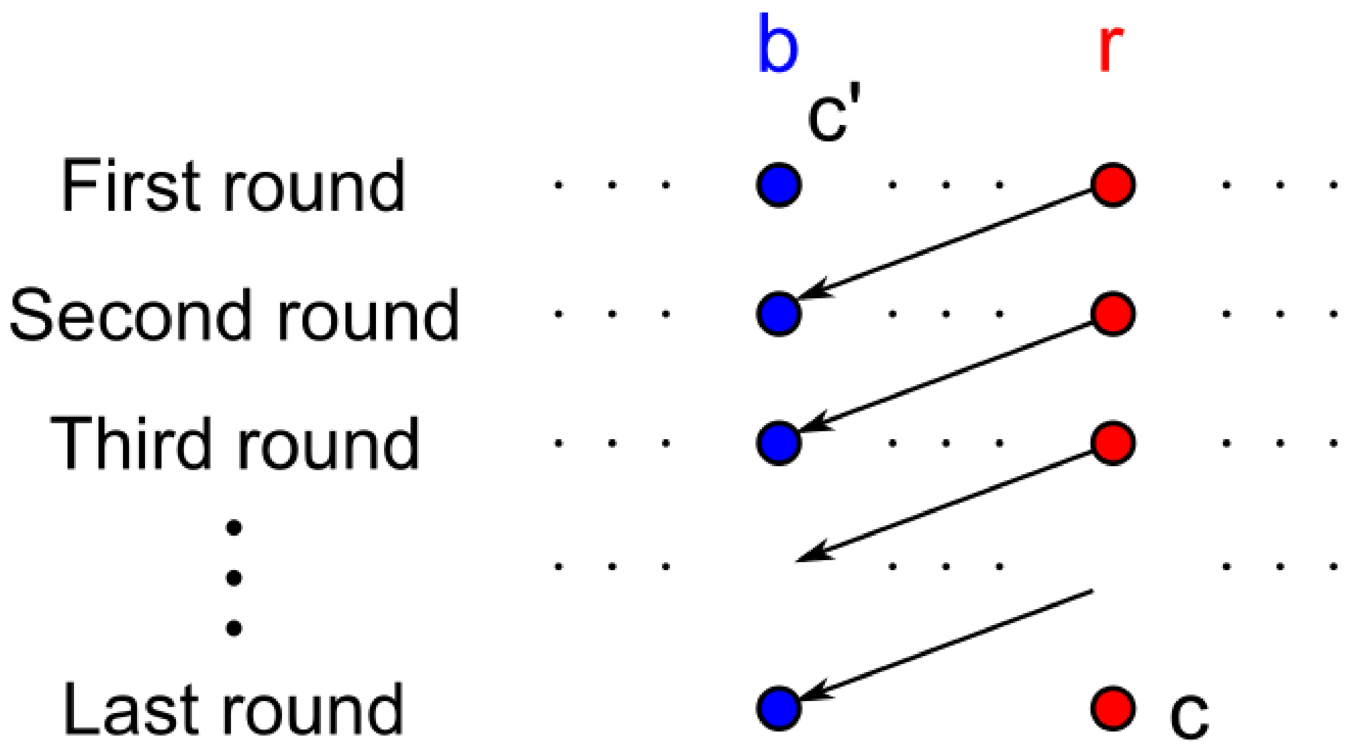

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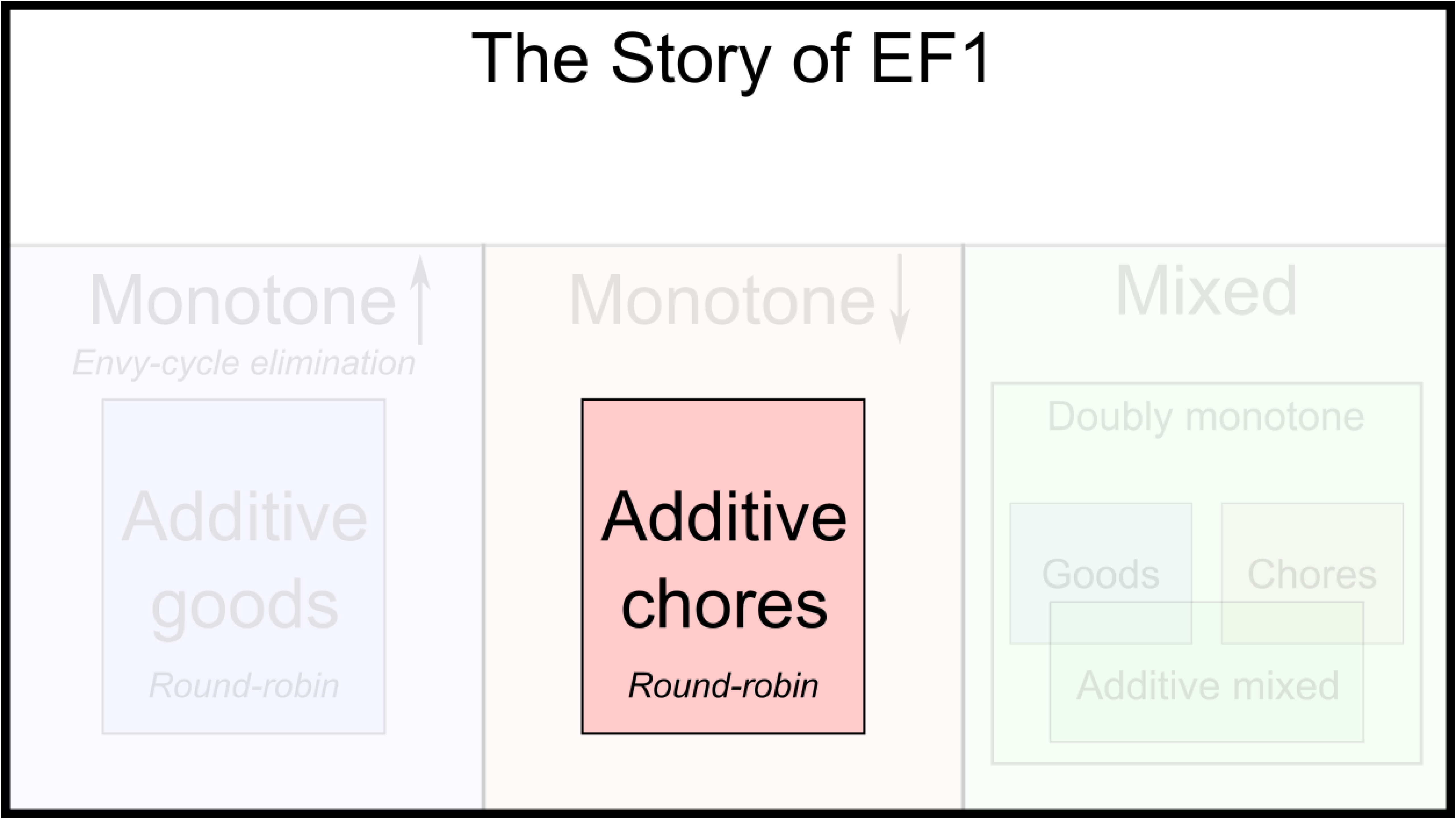
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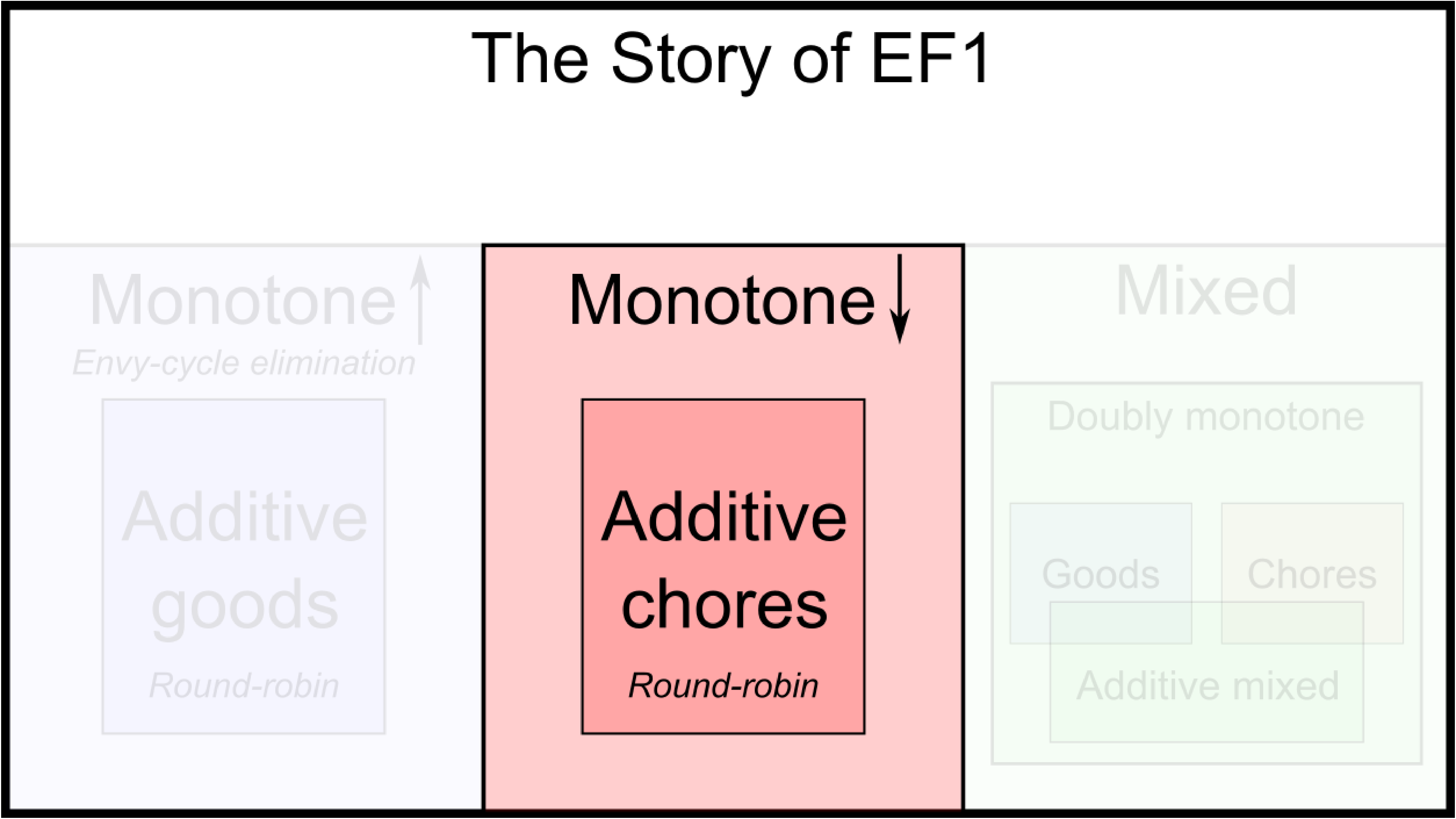

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Adapting envy-cycle elimination to chores

While there is an unallocated good

- If the envy graph has a source vertex, assign the good to that agent.
- Otherwise, resolve envy cycles until a source vertex shows up, and then assign the good to it.

Adapting envy-cycle elimination to chores

While there is an unallocated ~~good~~ chore

- If the envy graph has a ~~source~~ sink vertex, assign the ~~good~~ chore to that agent.
- Otherwise, resolve envy cycles until a ~~source~~ sink vertex shows up, and then assign the ~~good~~ chore to it.

Adapting envy-cycle elimination to chores

While there is an unallocated chore

- If the envy graph has a sink vertex, assign the chore to that agent.
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Adapting envy-cycle elimination to chores

While there is an unallocated chore

- If the envy graph has a sink vertex, assign the chore to that agent.
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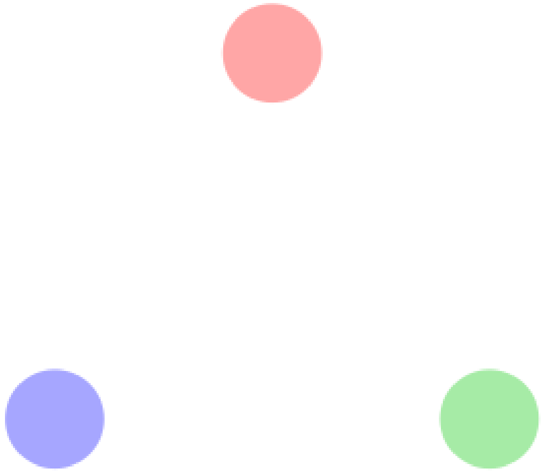
	A	B	C	D	E	F
	-1	-4	-2	-3	0	-1
	-2	-1	-2	-2	-3	-1
	-1	-3	-1	-1	-3	-10



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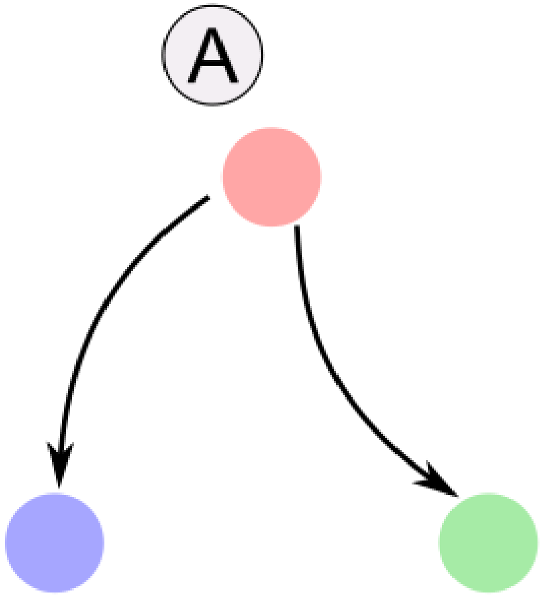
	-1	-3	-1	-1	-3	-10



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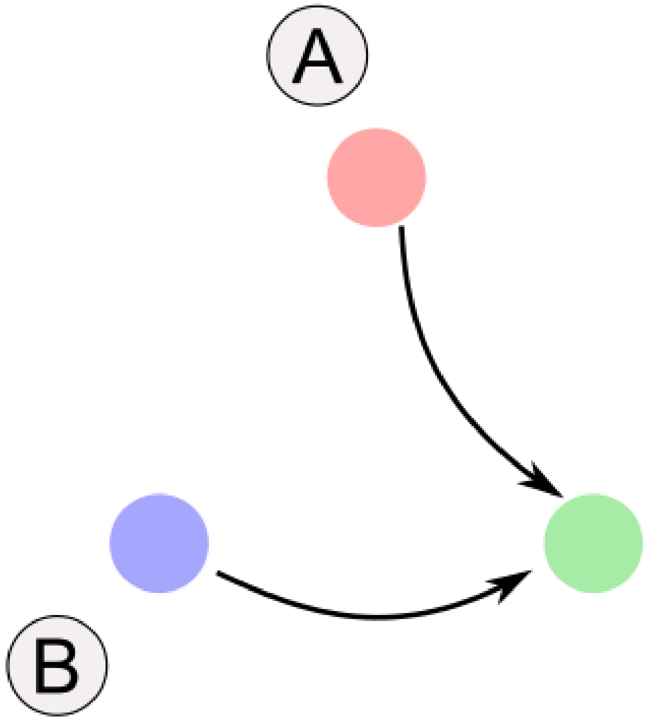
	-1	-3	-1	-1	-3	-10



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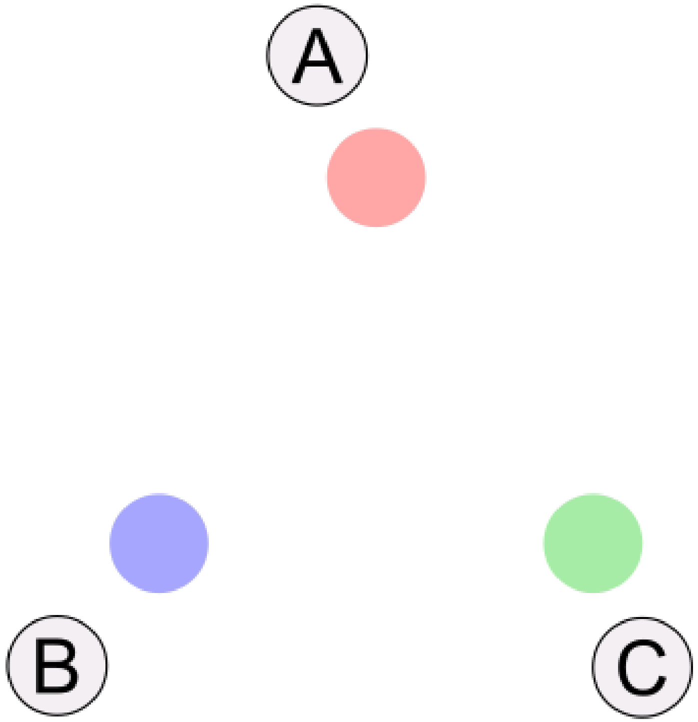
	-2	-1	-2	-2	-3	-1
	-1	-3	-1	-1	-3	-10



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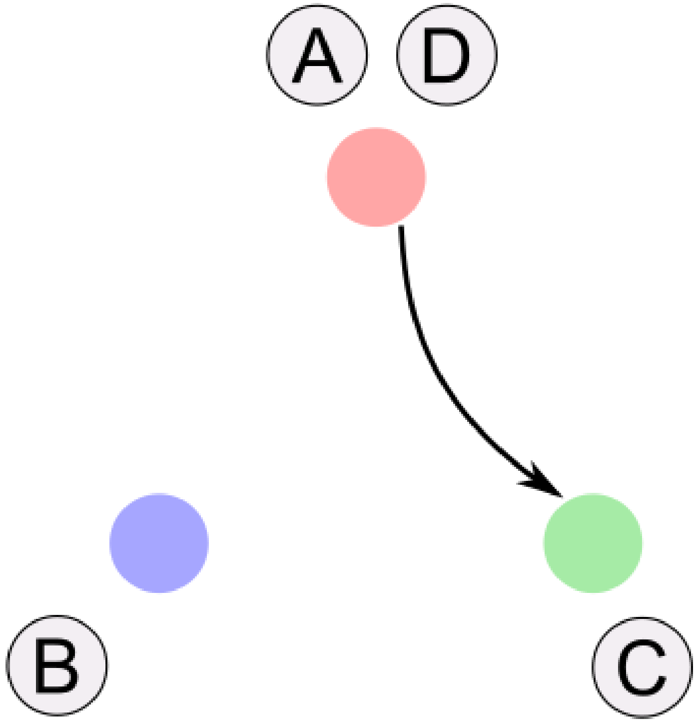
	-1	-3	-1	-1	-3	-10



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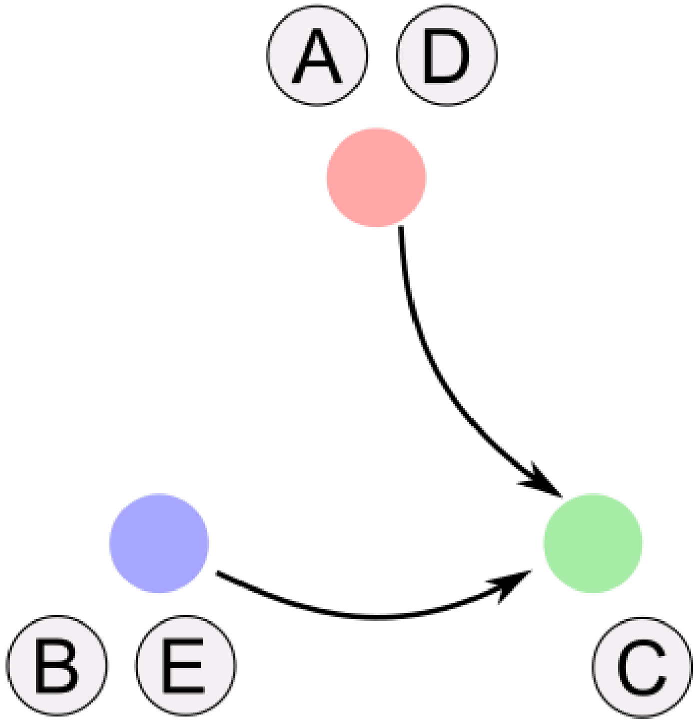
	-1	-3	-1	-1	-3	-10



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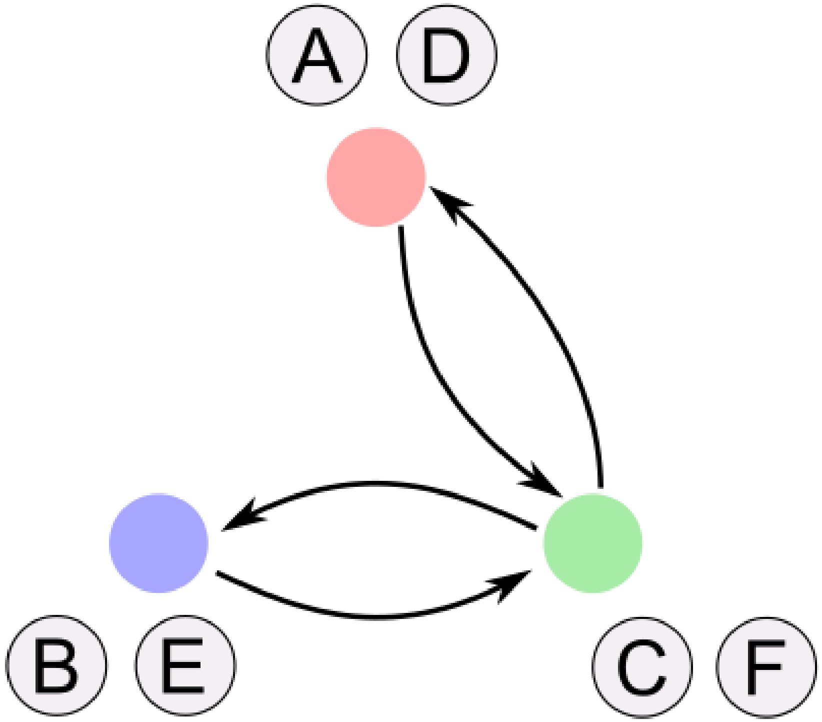

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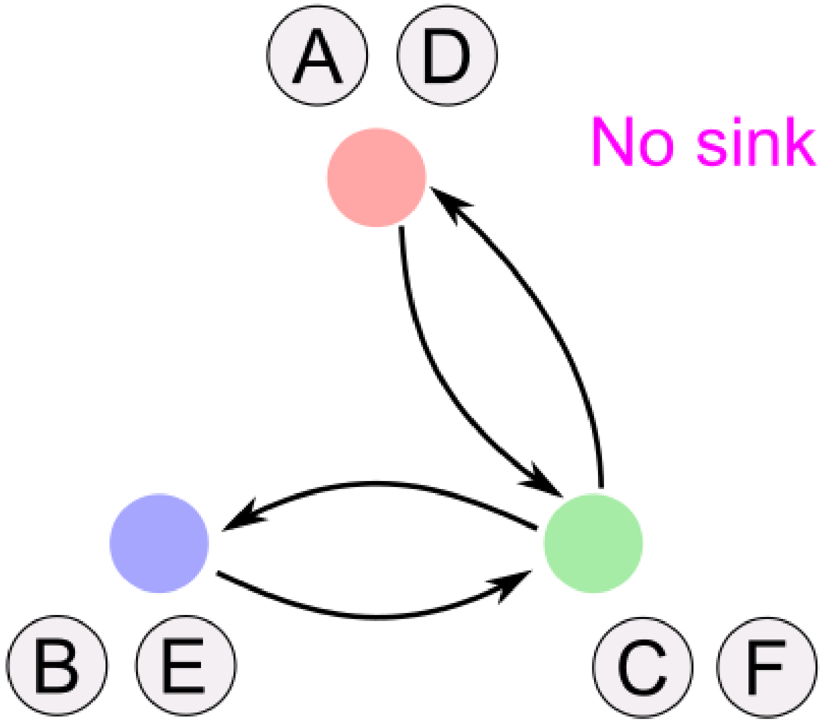

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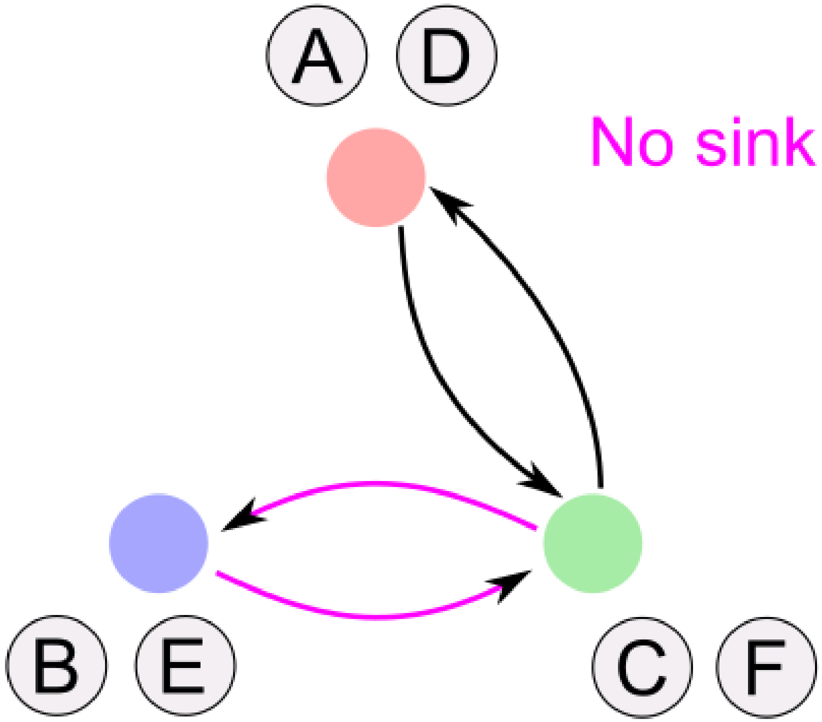
	-1	-3	-1	-1	-3	-10



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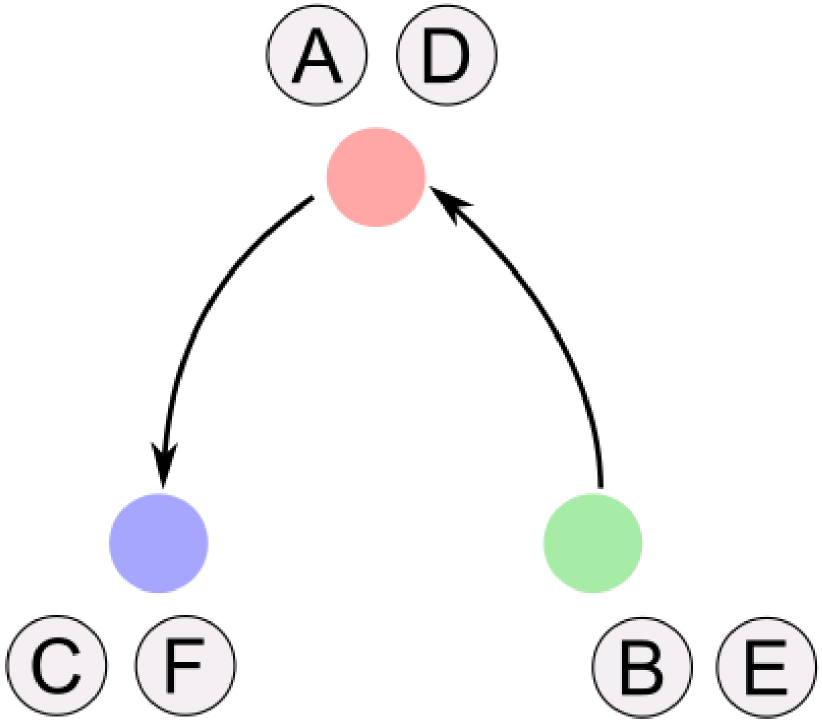
	A	B	C	D	E	F
	-1	-4	-2	-3	0	-1
	-2	-1	-2	-2	-3	-1
	-1	-3	-1	-1	-3	-10

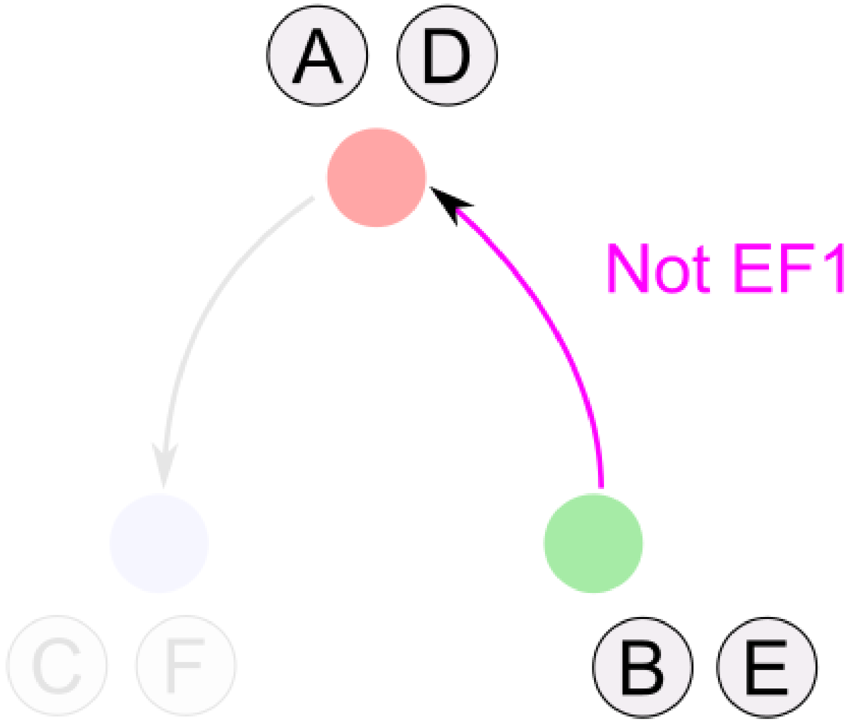


Adapting envy-cycle elimination to chores

While there is an unallocated chore

- If the envy graph has a sink vertex, assign the chore to that agent.
- Otherwise, resolve envy cycles until a sink vertex shows up, and then assign the chore to it.

	A	B	C	D	E	F
	-1	-4	-2	-3	0	-1
	-2	-1	-2	-2	-3	-1
	-1	-3	-1	-1	-3	-10



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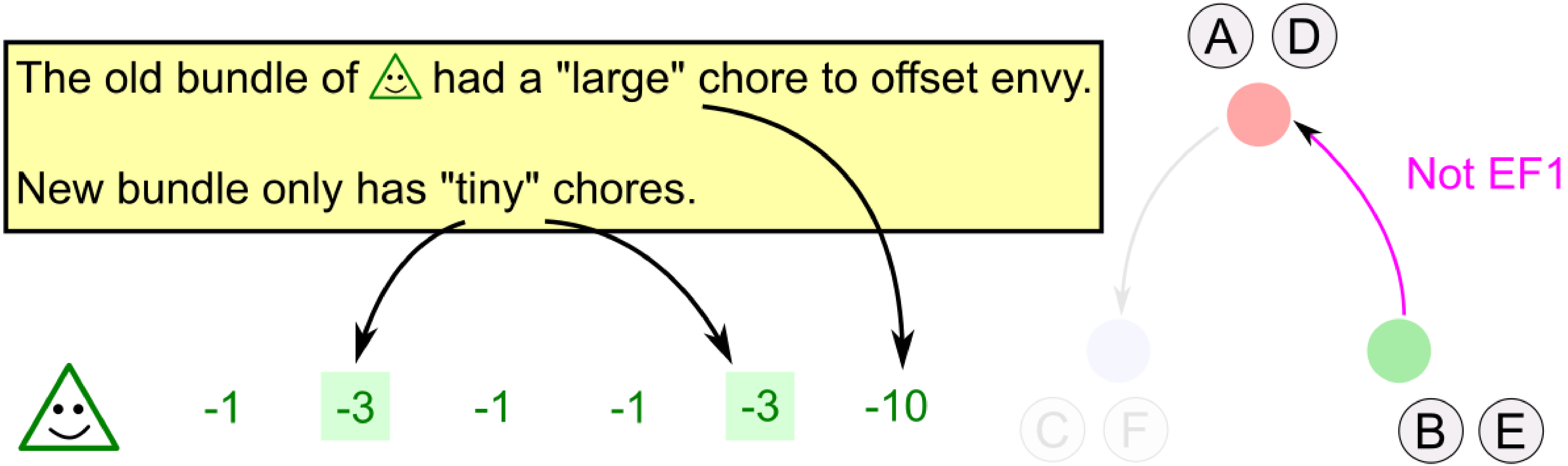

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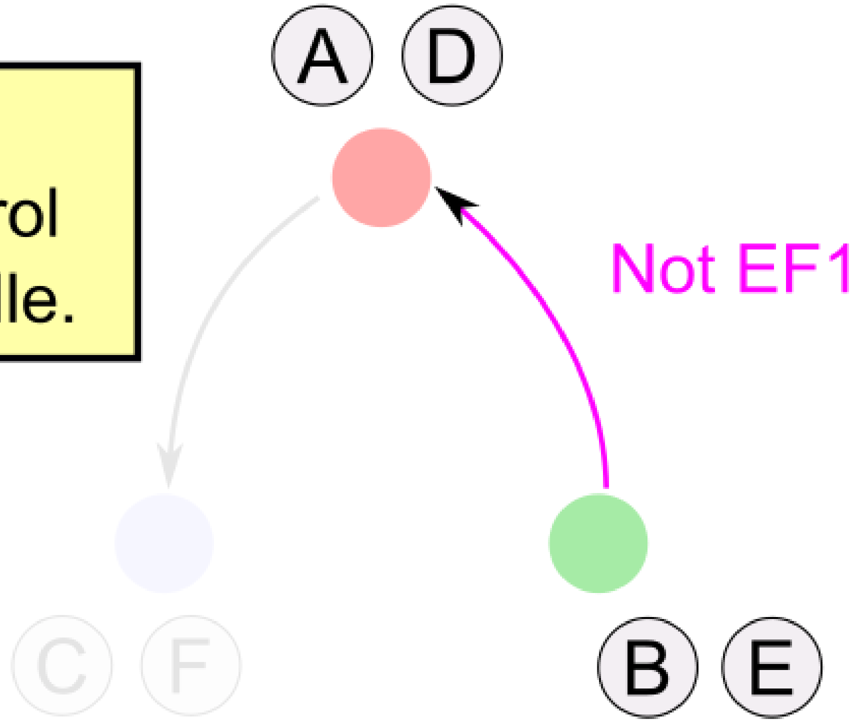
Source of the problem

Resolving arbitrary envy cycles gives us no control over the size of individual chores in the new bundle.



-1 -3 -1 -1 -3 -10

Adapting envy-cycle elimination to chores

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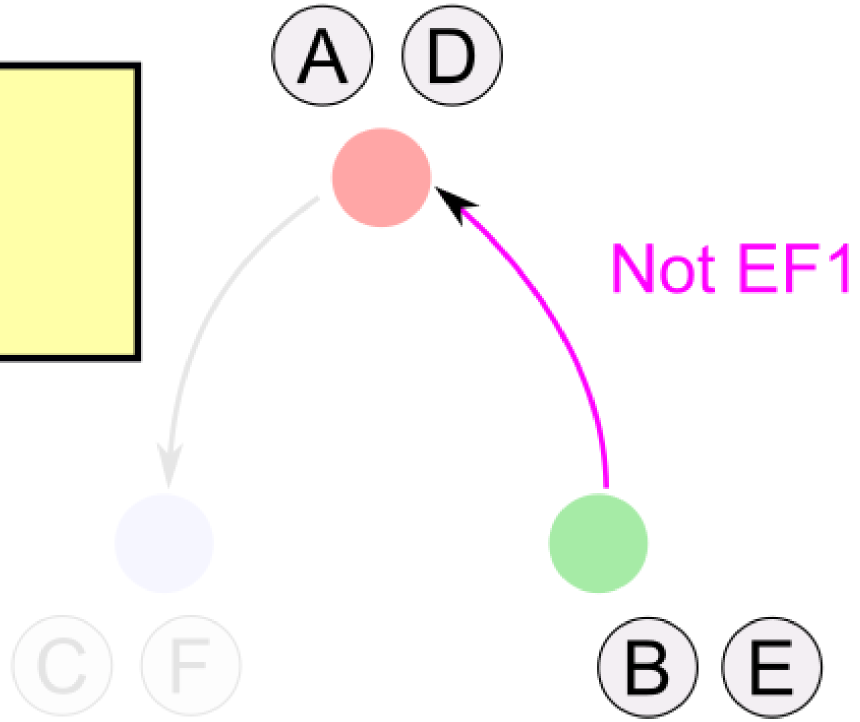
Solution

Resolve top-trading envy cycle

Each agent points to its *favorite* envied bundle

-1 -3 -1 -1 -3 -10



Top-trading envy-cycle elimination

[Bhaskar, Sricharan, and Vaish, *APPROX* 2021]

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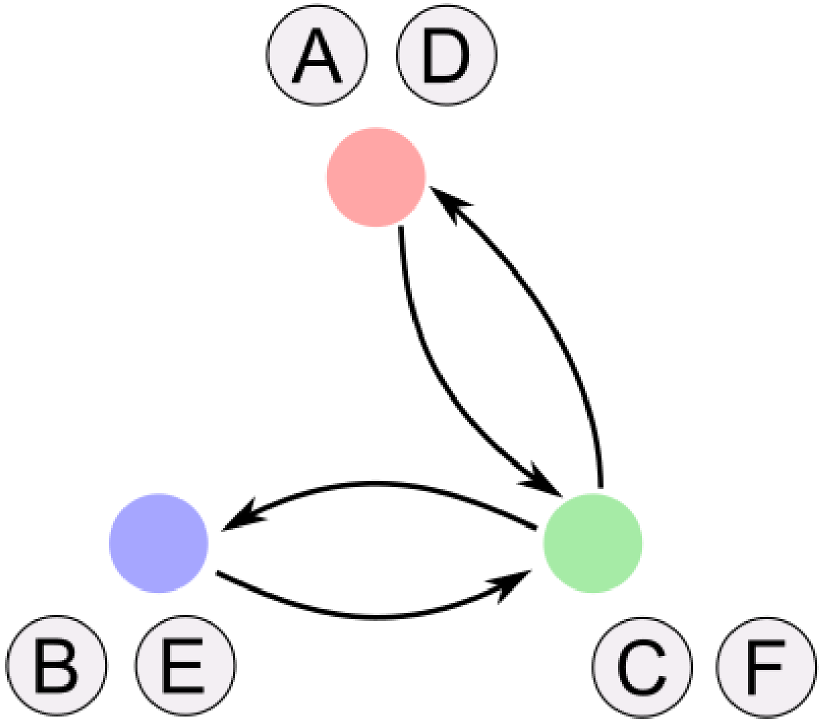

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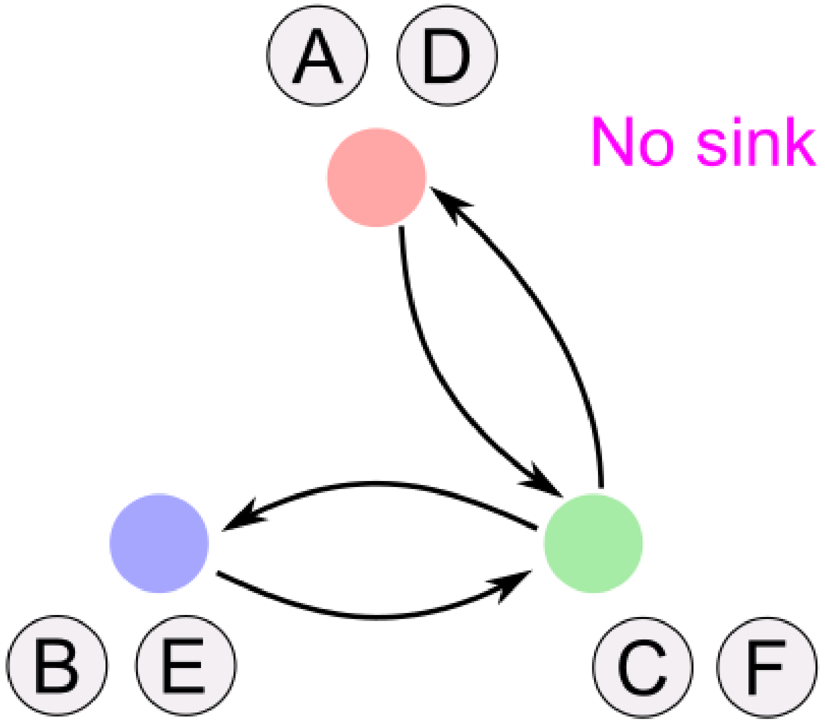

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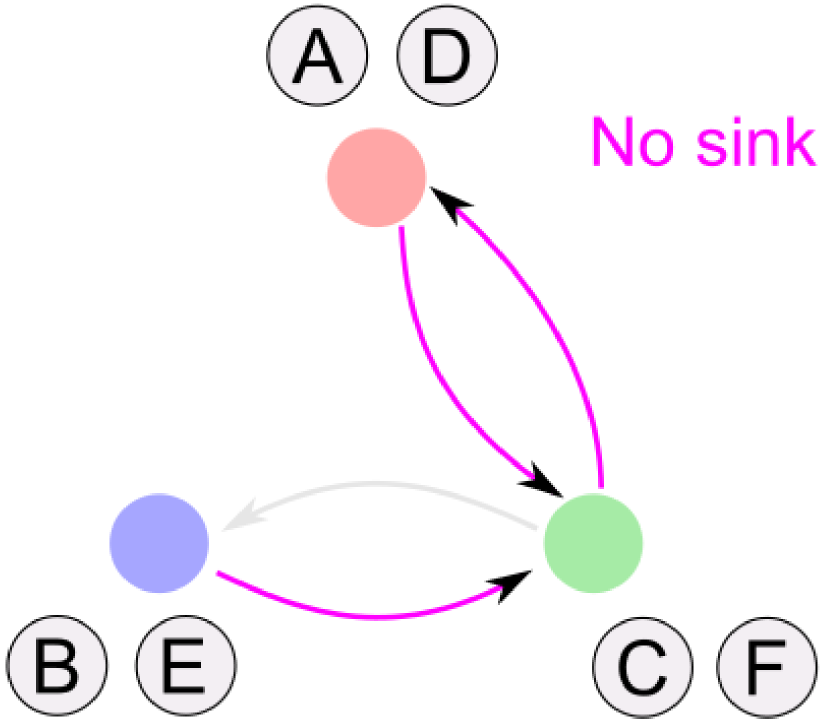

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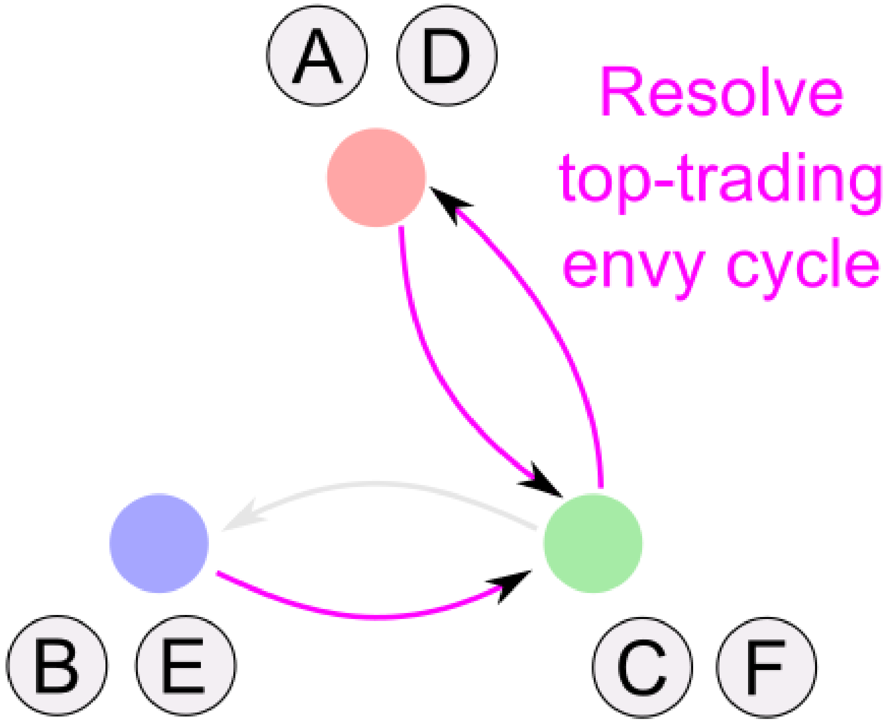

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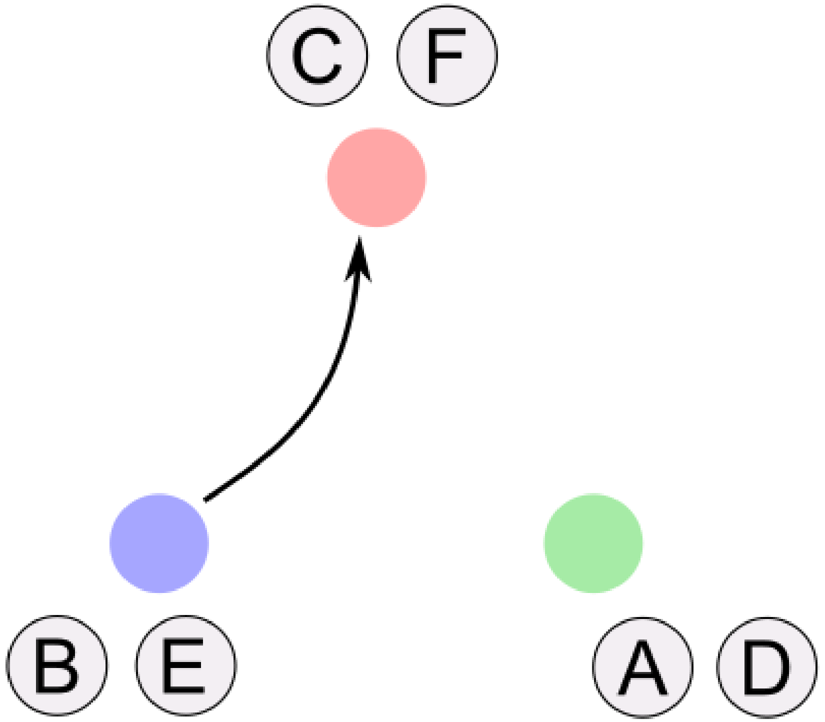

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⇒ There is a cycle of "most envied" edges

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Why does top-trading envy cycle algorithm satisfy EF1?

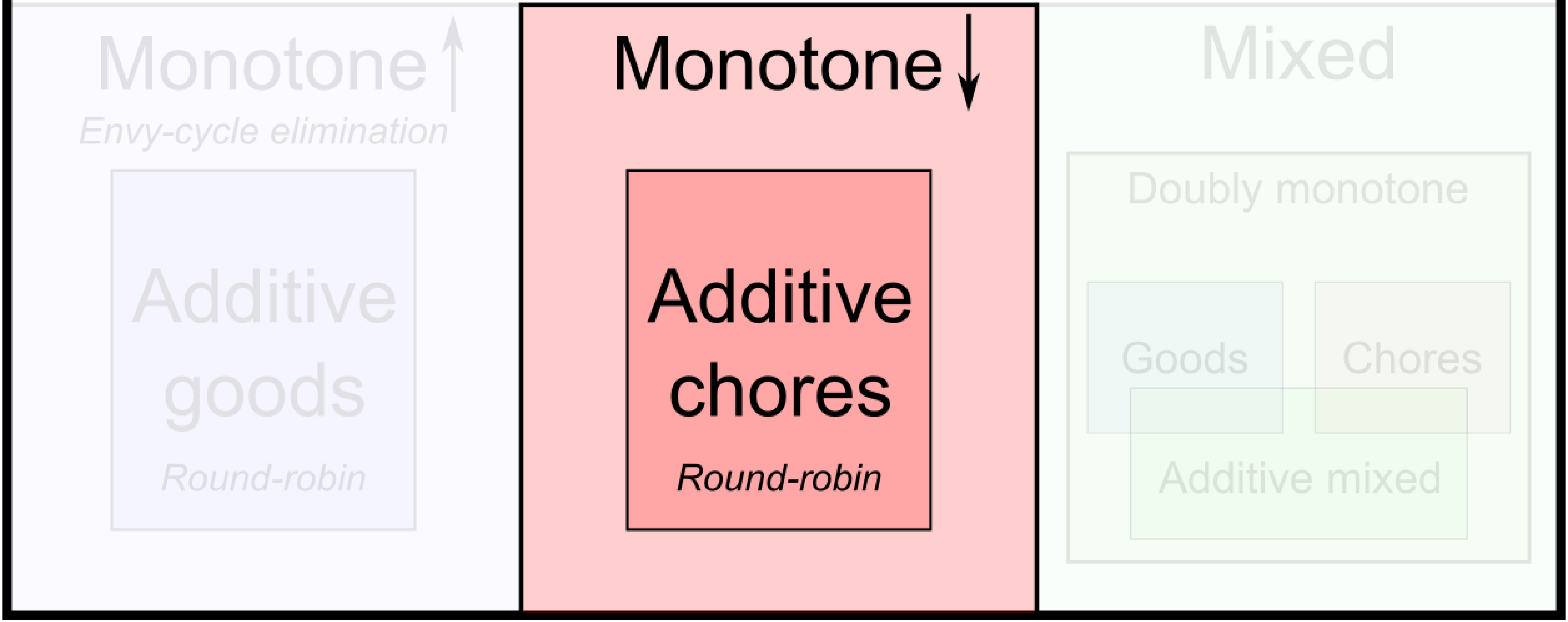
Every vertex in the top-trading cycle becomes envy-free.

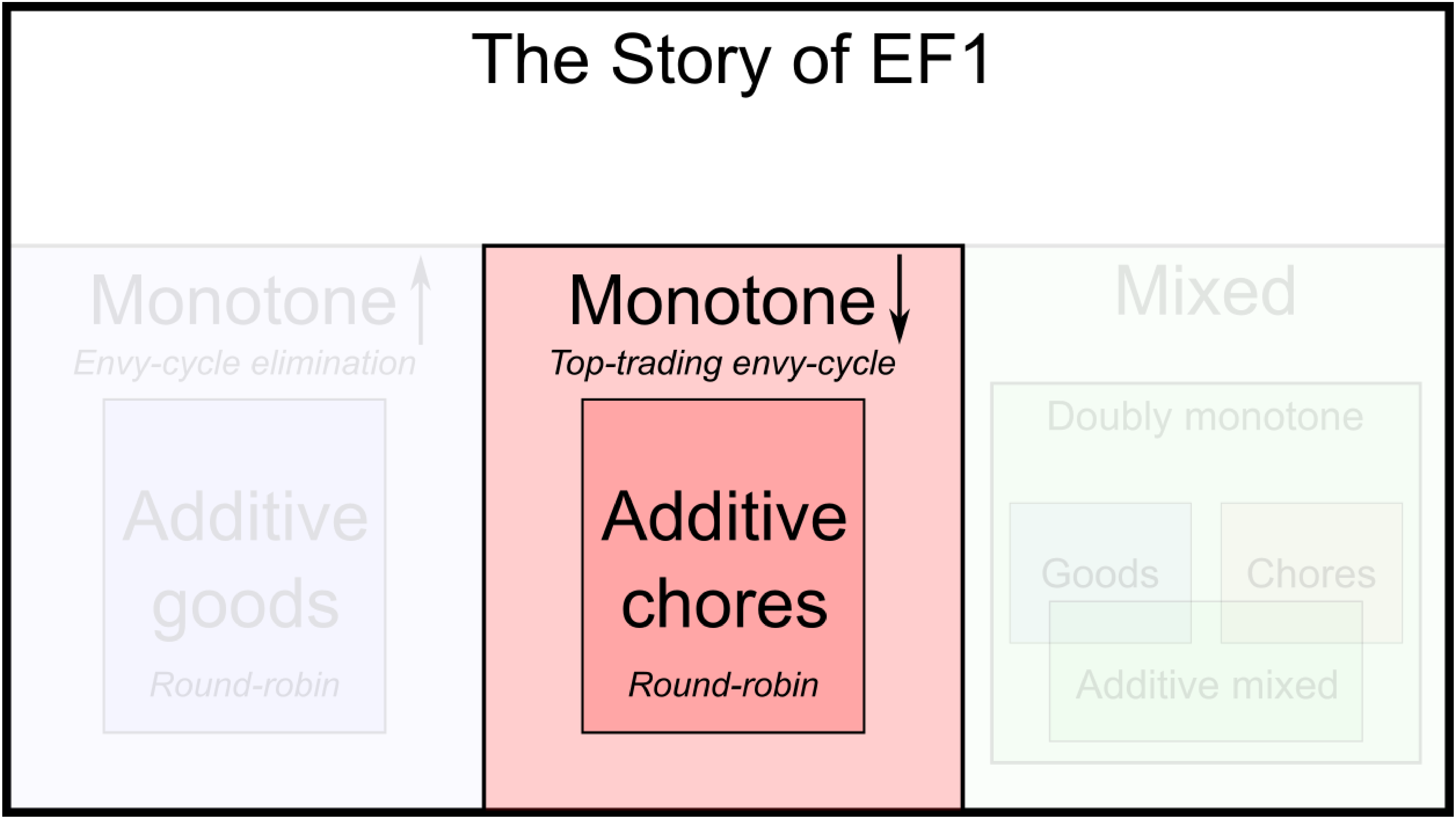
The problem of "new bundle with tiny chores" does not arise.

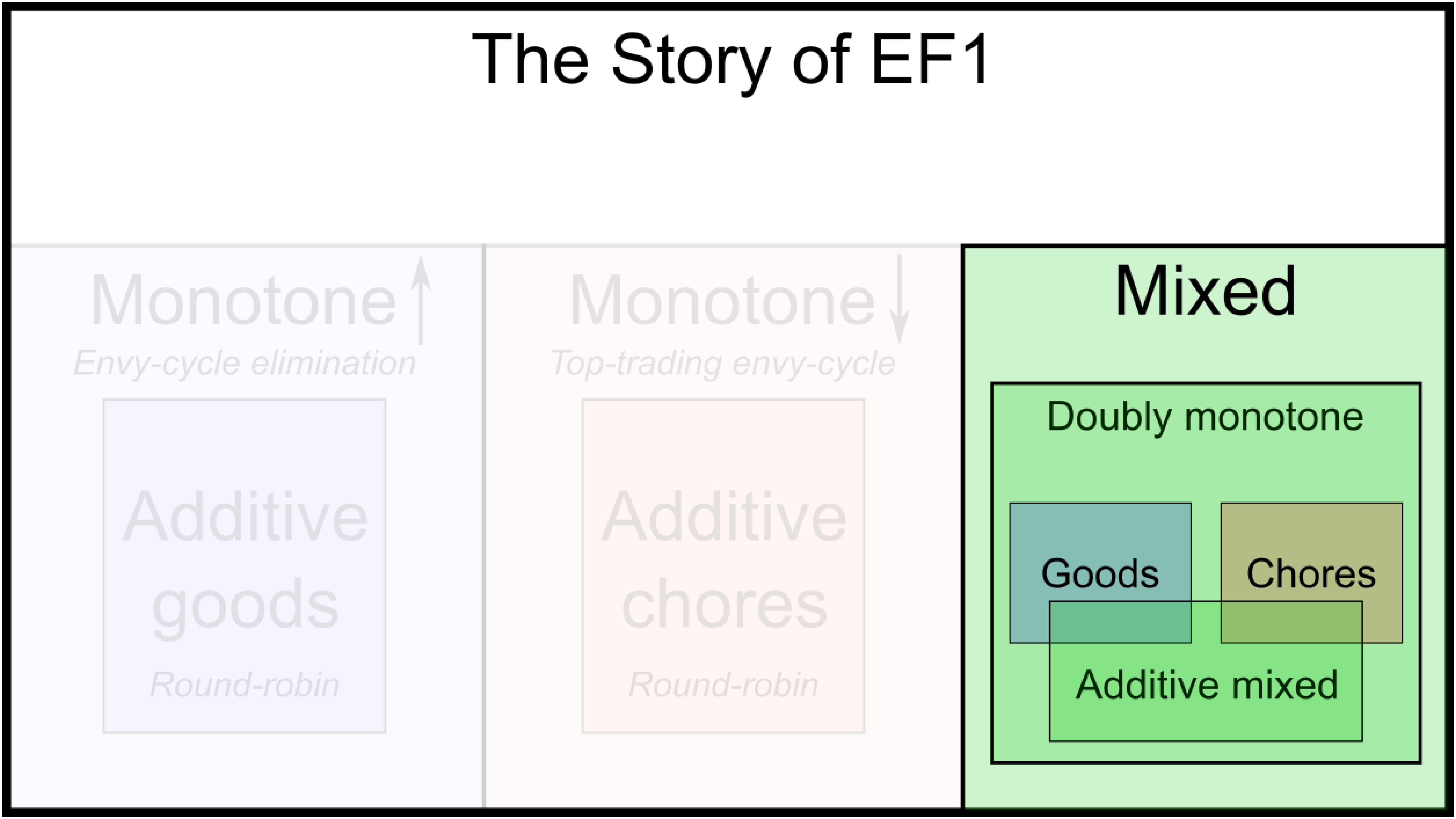
[Bhaskar, Sricharan, and Vaish, *APPROX* 2021]

For monotone chores, the allocation computed by the top-trading envy-cycle elimination algorithm satisfies EF1.

The Story of EF1




The Story of EF1
Monotone↑
Envy-cycle elimination
Additive goods
Round-robin
Monotone↓
Top-trading envy-cycle
Additive chores
Round-robin
Mixed
Doubly monotone
Goods
Chores
Additive mixed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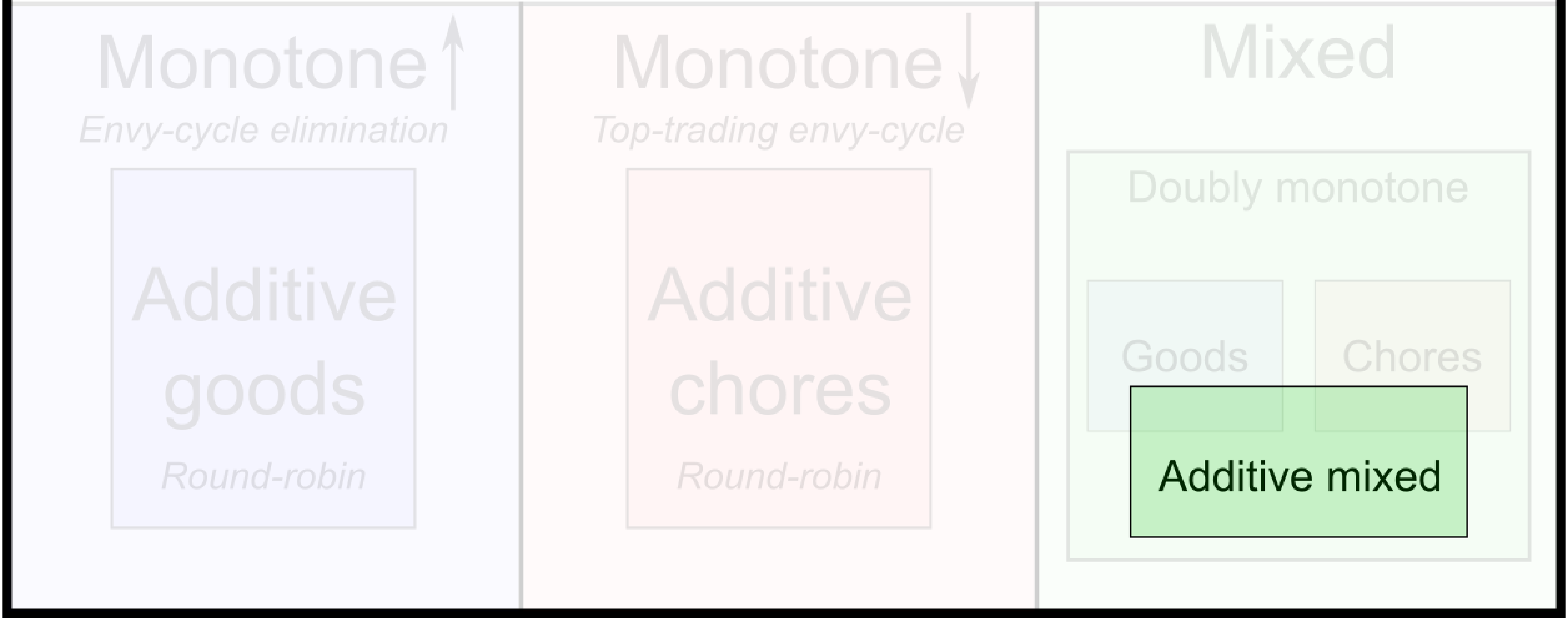
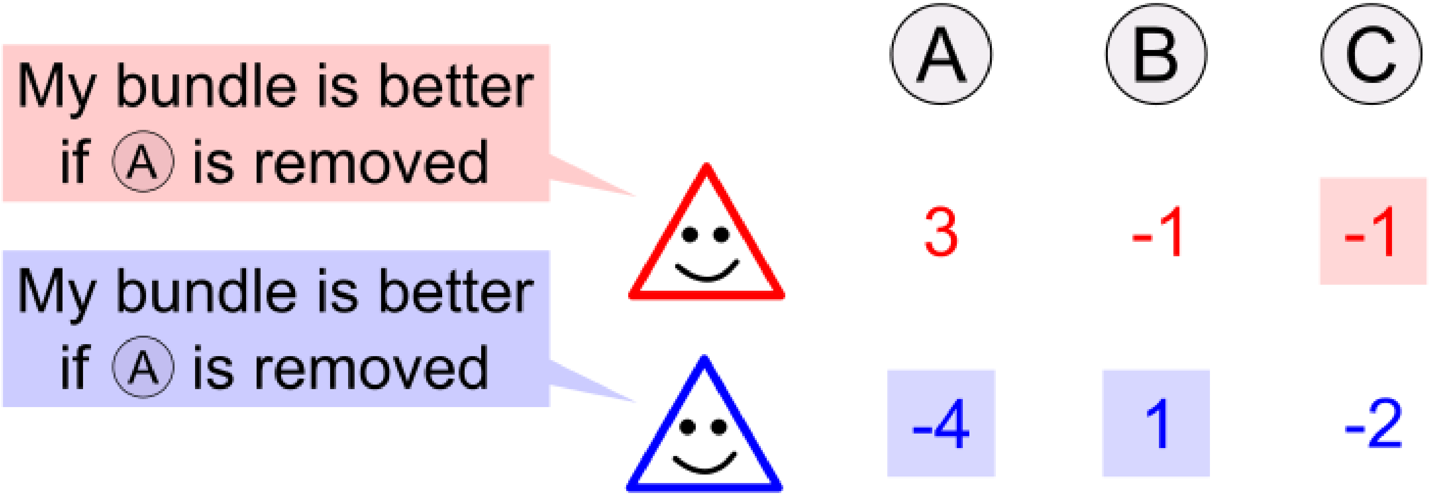


Envy-Freeness Up To One Item

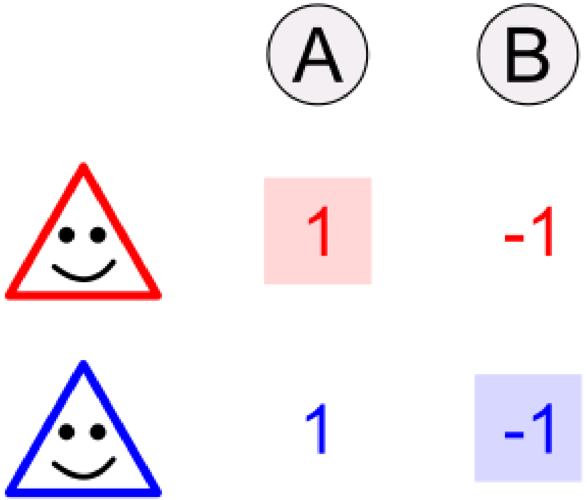
[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

Eliminate envy by removing some "good" in the envied bundle
or some "chore" in the envious agent's bundle.



Allocation $A = (A_1, \ldots, A_n)$ is EF1 if for every pair of agents i, k, there exists an item $j \in A_i \cup A_k$ s.t. $v_i(A_i \setminus \{j\}) \geq v_i(A_k \setminus \{j\})$.

For goods+chores, naive round-robin fails EF1.



Double round-robin algorithm

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

Double round-robin algorithm

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

Partition the items into two sets: positive and negative

Positive: items with strictly positive value for at least one agent

(considered to be a "good" by at least one agent)

Negative: all other items

(considered a "chore" by all agents)

Double round-robin algorithm

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

	A	B	C	D	E
	-4	-1	-2	2	-4
	0	-1	-5	-2	-1
	-4	-2	-5	0	2

Double round-robin algorithm

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

	A	B	C	D	E
	-	-	-	+	+
	-4	-1	-2	2	-4
	0	-1	-5	-2	-1
	-4	-2	-5	0	2

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[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

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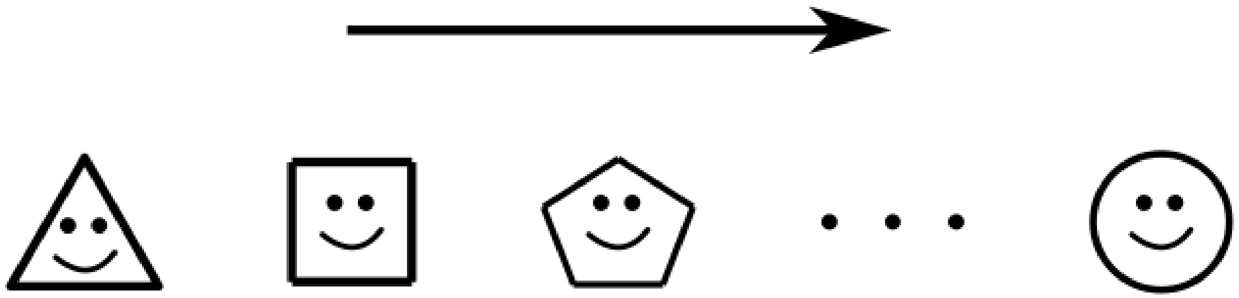
allocate negative items in this order



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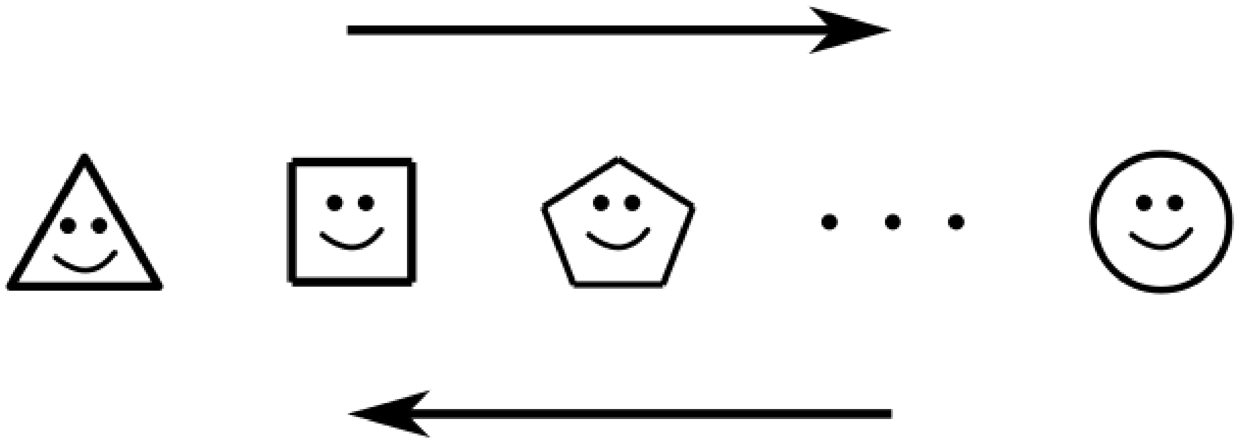

and positive items in the opposite order

Double round-robin algorithm

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

No. of negative items is an integer multiple of n (add zero valued items)

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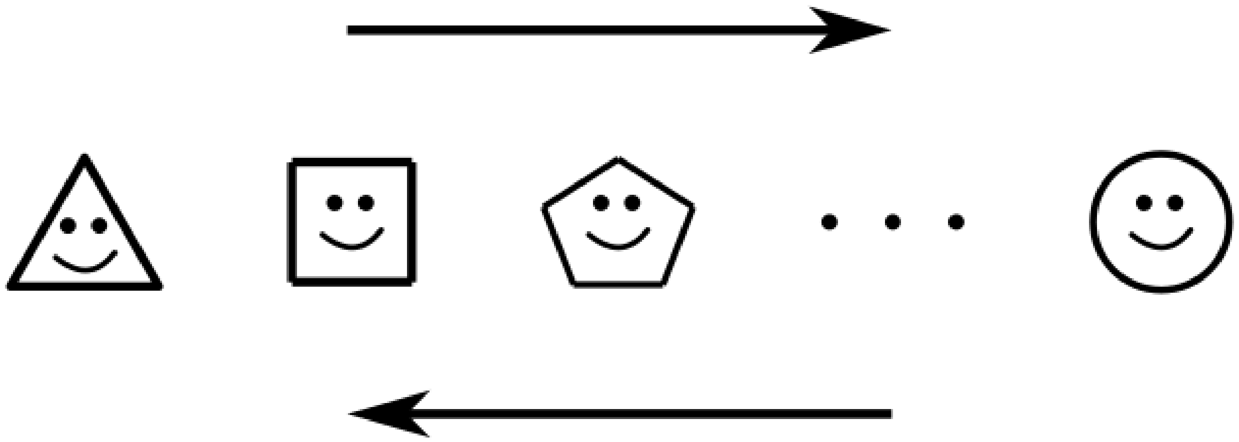

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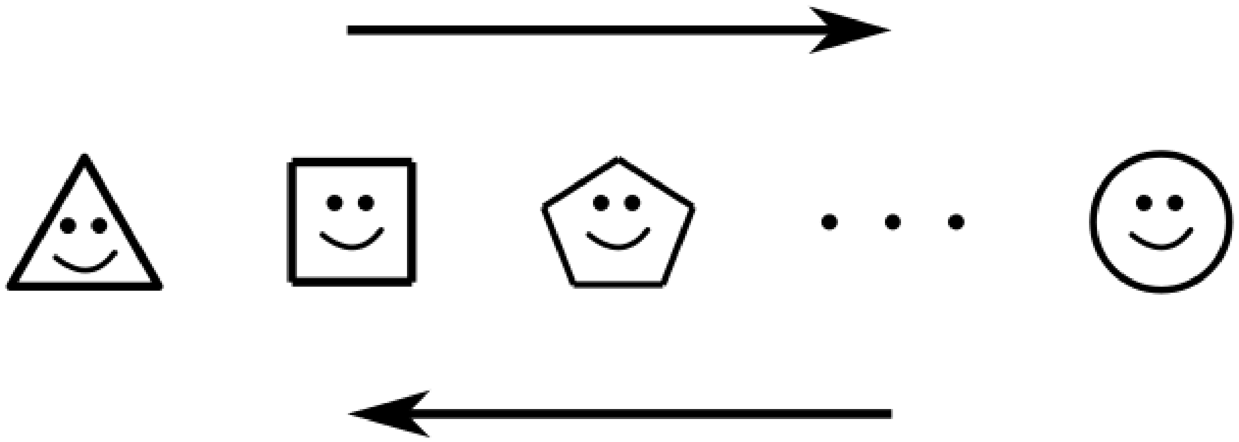

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Picking with skipping

Double round-robin algorithm

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

	-	-	-	+	+
	A	B	C	D	E
	-4	-1	-2	2	-4
	0	-1	-5	-2	-1
	-4	-2	-5	0	2

Double round-robin algorithm

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]

	-	-	-	+	+
	A	B	C	D	E
	-4	-1	-2	2	-4
	0	-1	-5	-2	-1
	-4	-2	-5	0	2

Double round-robin algorithm

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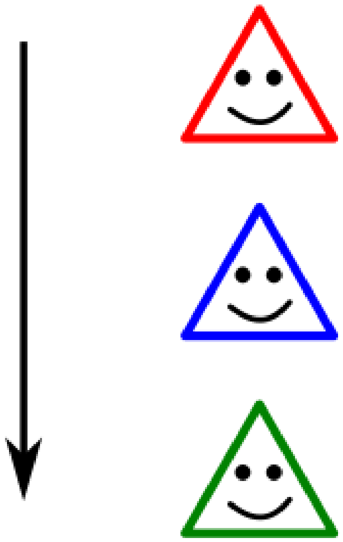

	-	-	-	+	+
	A	B	C	D	E
	-4	-1	-2	2	-4
	0	-1	-5	-2	-1
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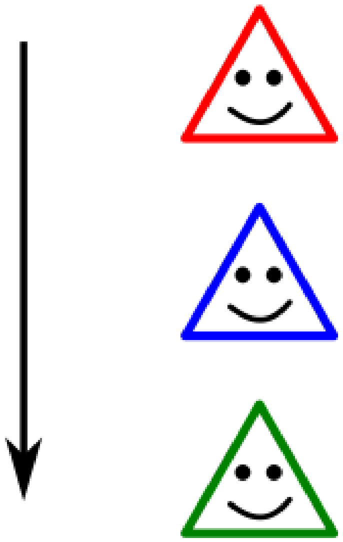

-	-	-	+	+
A	B	C	D	E
-4	-1	-2	2	-4
0	-1	-5	-2	-1
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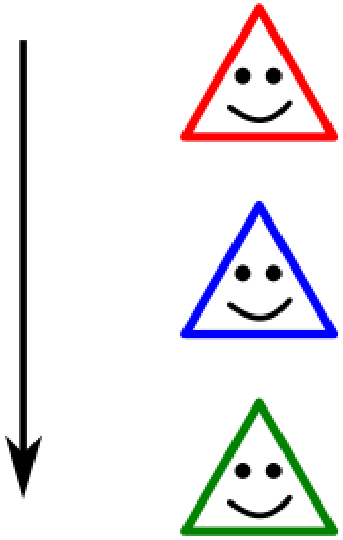

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	-	-	-	+	+
	-4	-1	-2	2	-4
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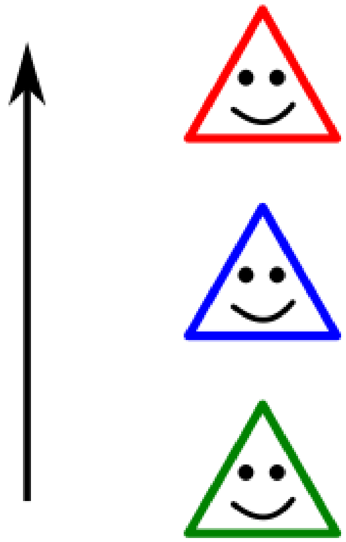

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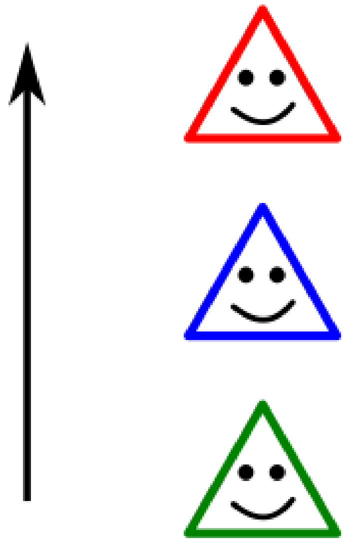
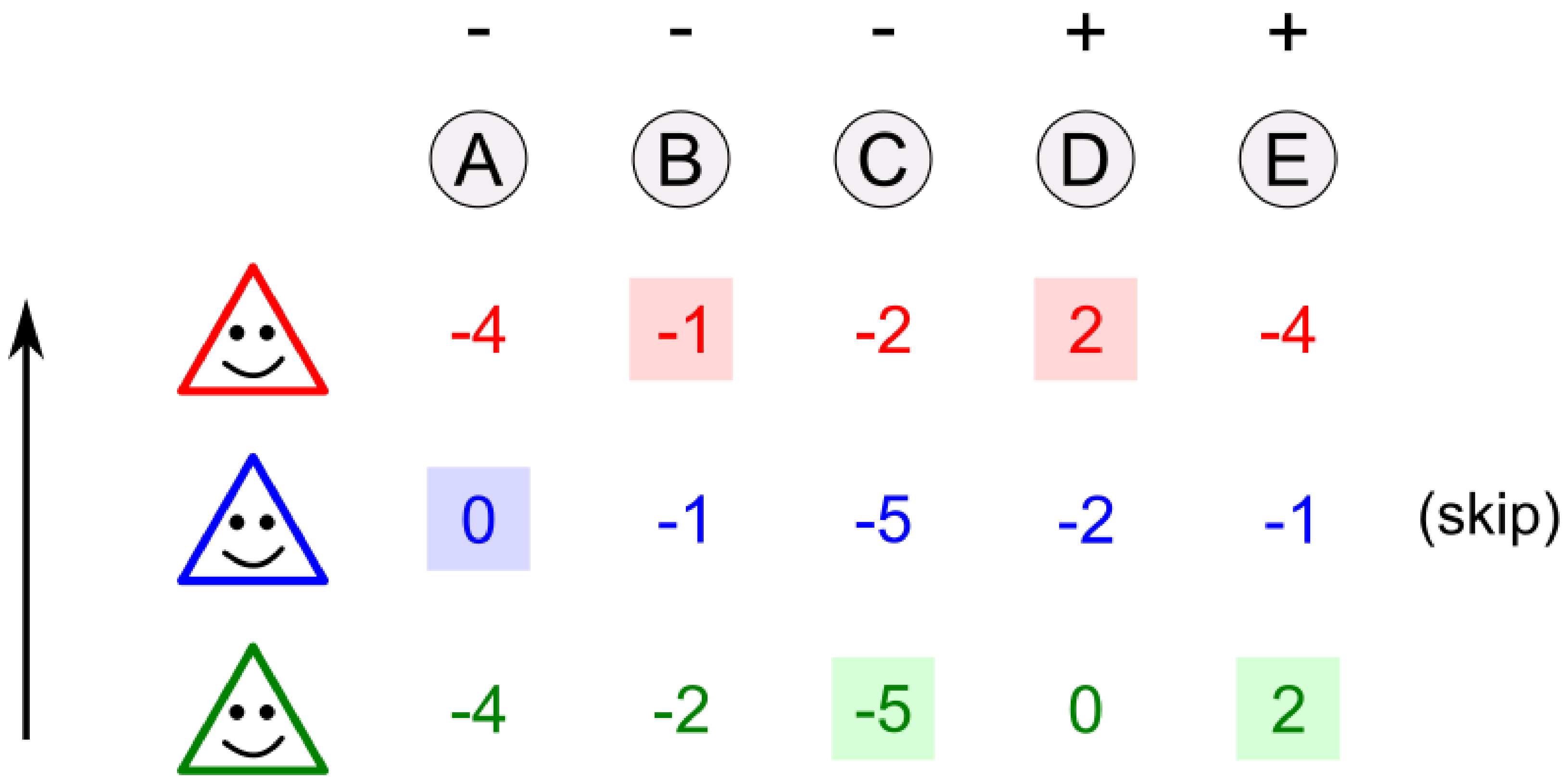


	A	B	C	D	E	
	-	-	-	+	+	
red agent	-4	-1	-2	2	-4	
blue agent	0	-1	-5	-2	-1	(skip)
green agent	-4	-2	-5	0	2	

Double round-robin algorithm

[Aziz, Caragiannis, Igarashi, and Walsh; *IJCAI* 2019; *JAAMAS* 2022]



Why does double round-robin algorithm satisfy EF1?

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Fix a pair of agents (r,b). Analyze envy of r towards b.

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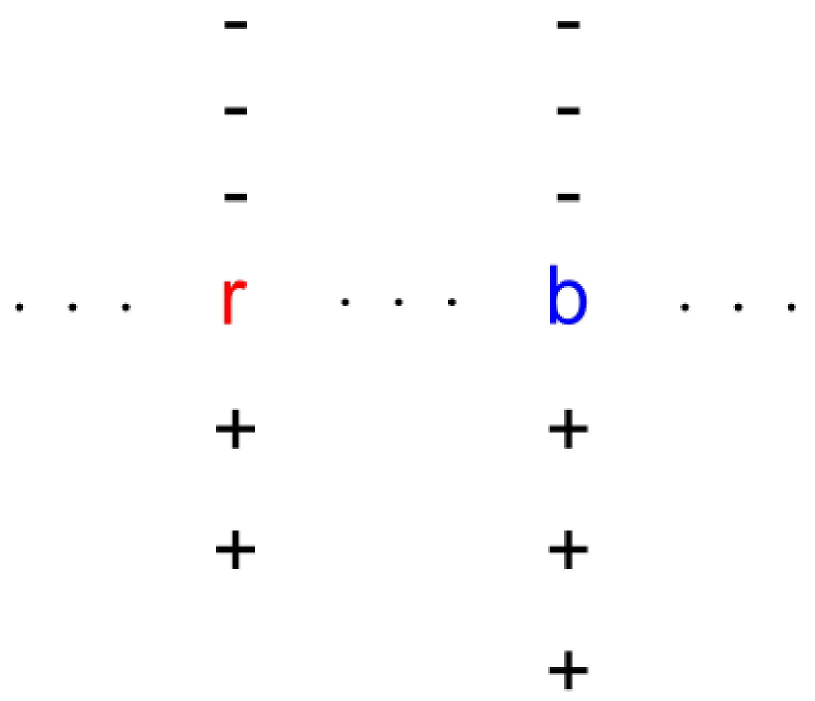

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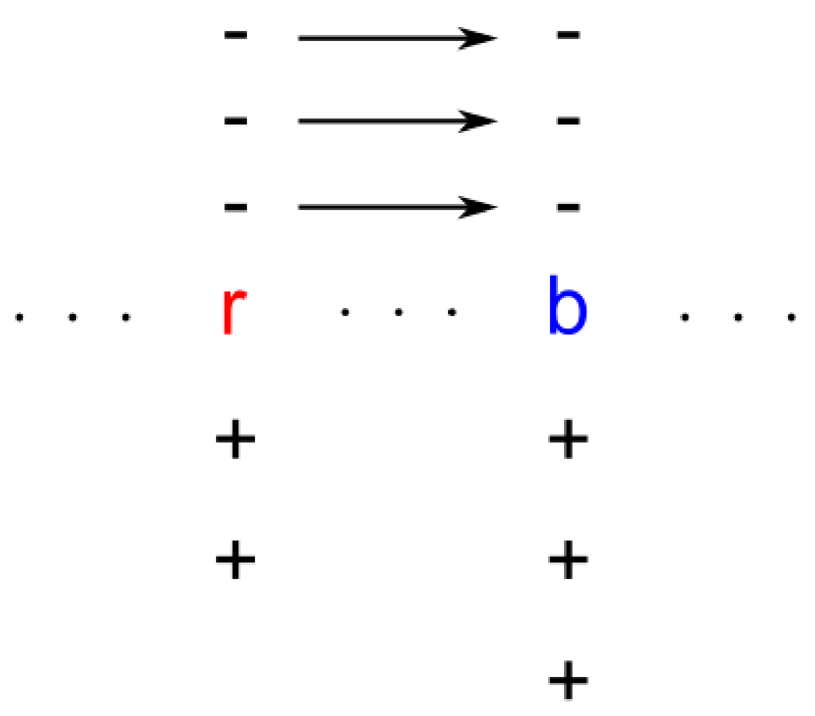

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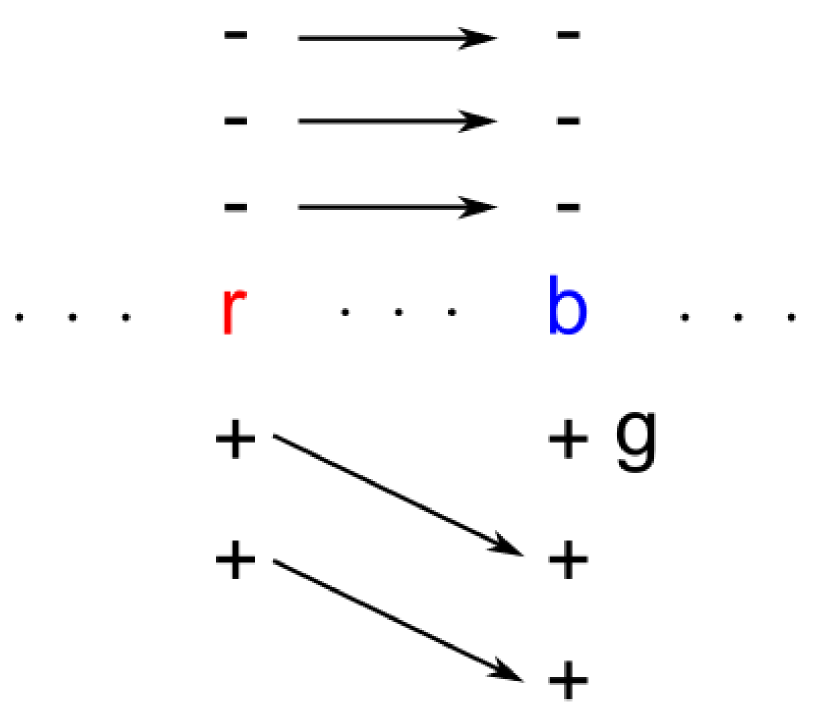

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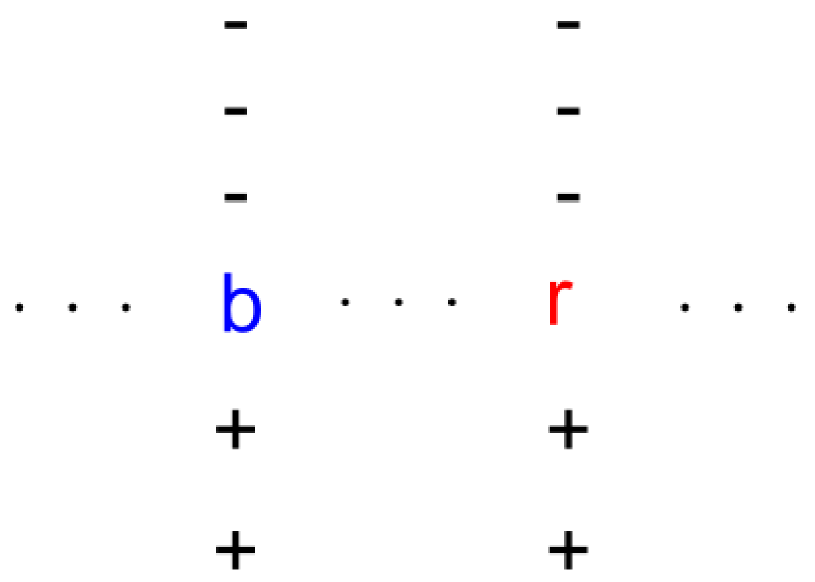

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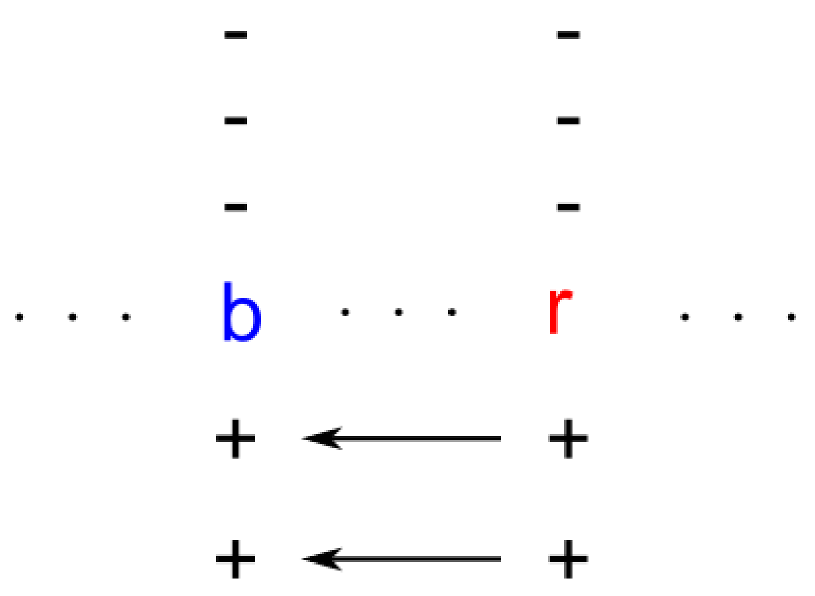

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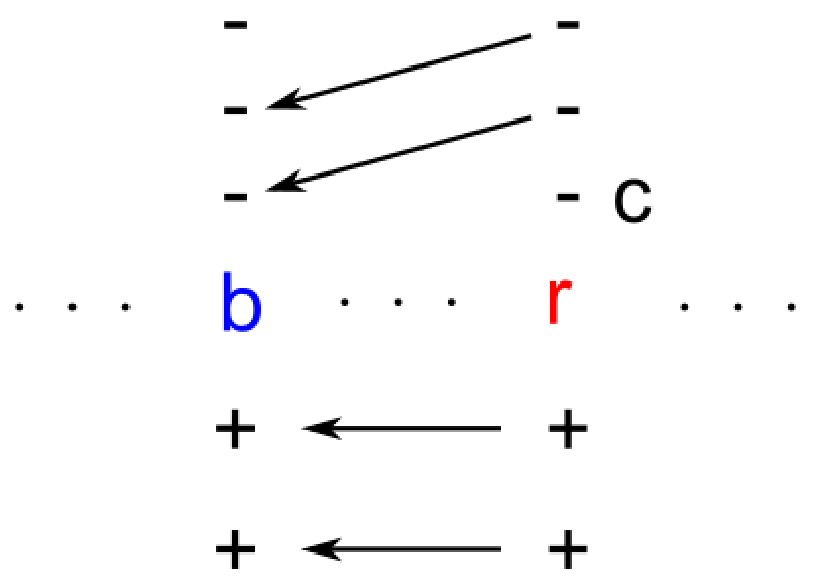

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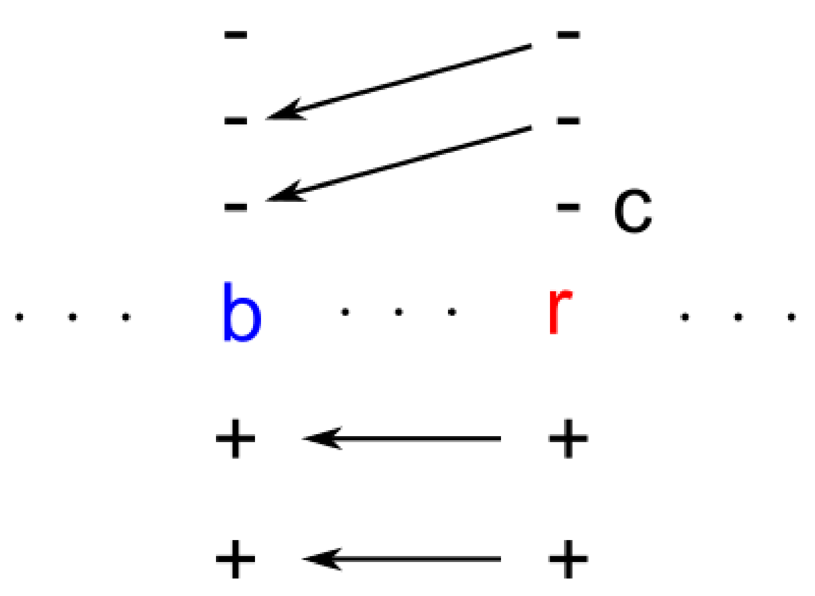
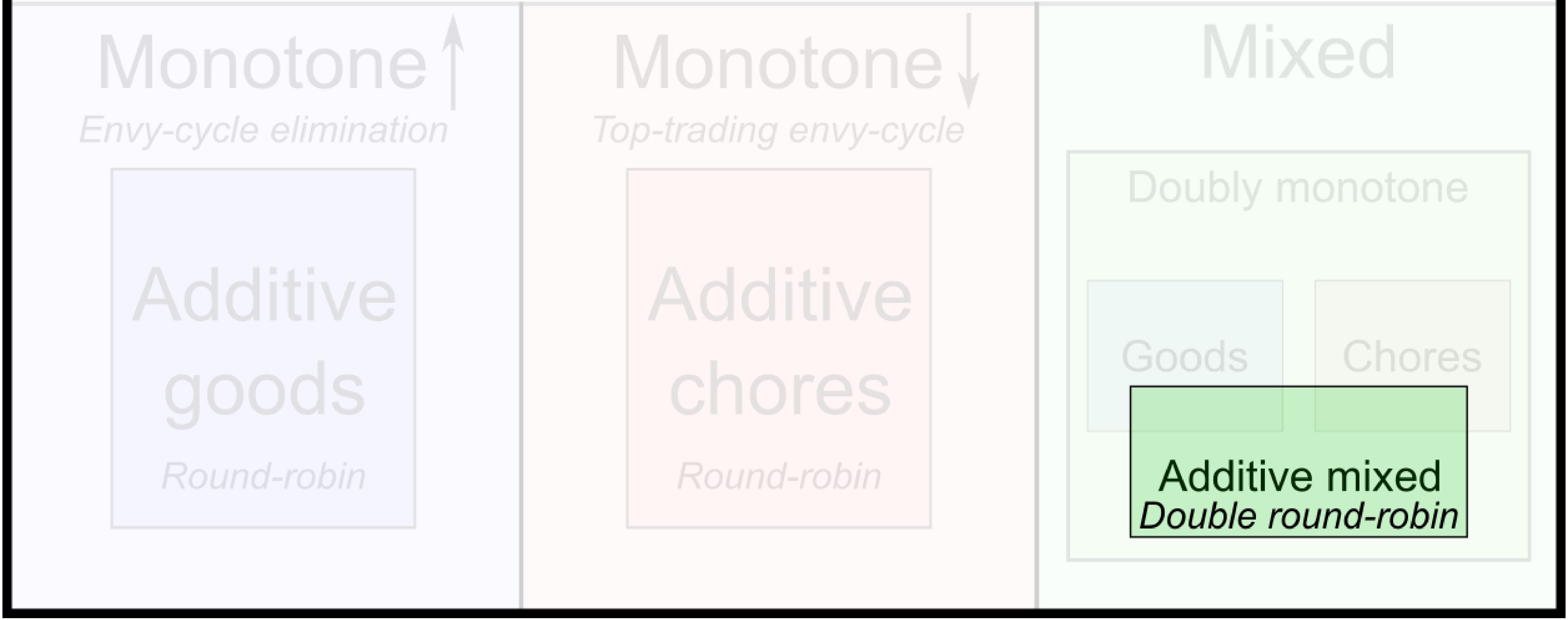
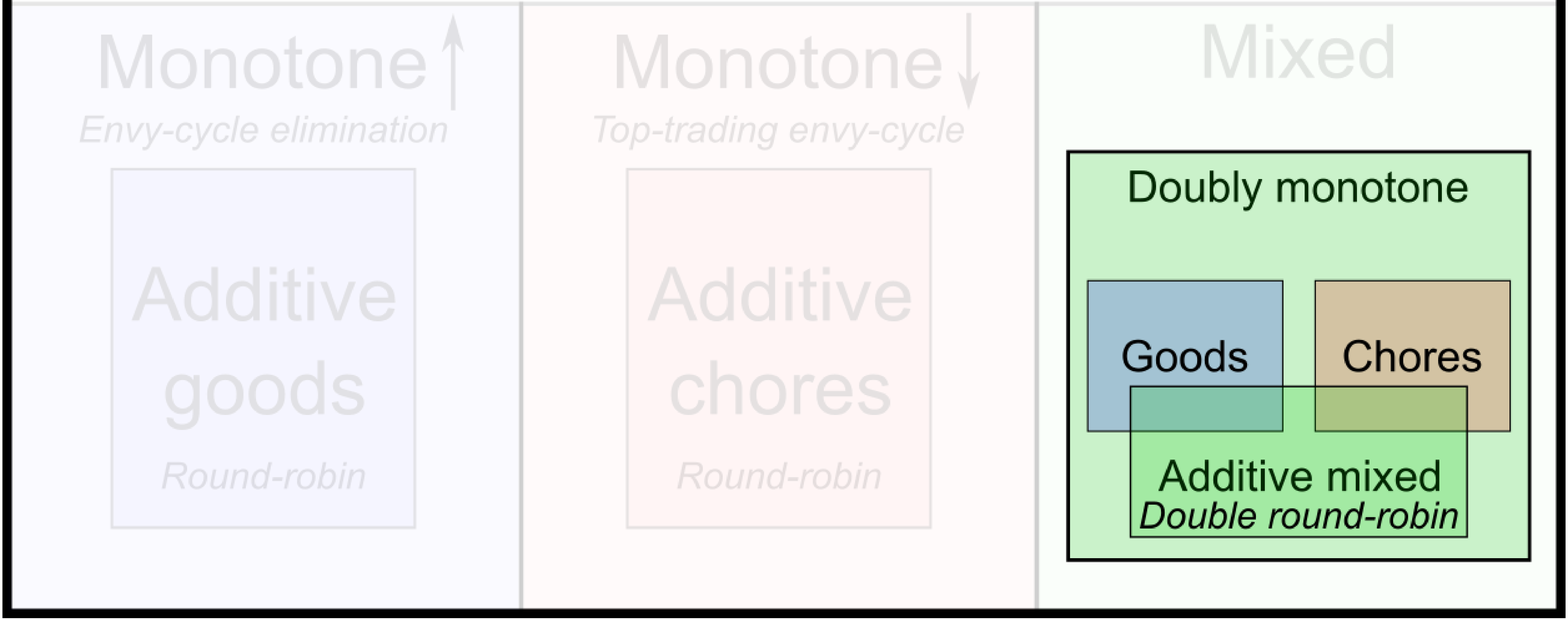


The Story of EF1



The Story of EF1



Doubly Monotone Valuations

Doubly Monotone Valuations

Each agent can partition the items into "goods" and "chores".

marginal ≥ 0

marginal ≤ 0

Doubly Monotone Valuations

Each agent can partition the items into "goods" and "chores".

marginal ≥ 0

marginal ≤ 0

	A	B	C	D	E
	-	-	-	+	+
	-	+	+	+	+
	-	+	-	+	-

EF1 for Doubly Monotone Valuations

Partition the items into two sets: positive and negative

Positive: items considered "good" by at least one agent

Negative: items considered "chore" by everyone

EF1 for Doubly Monotone Valuations

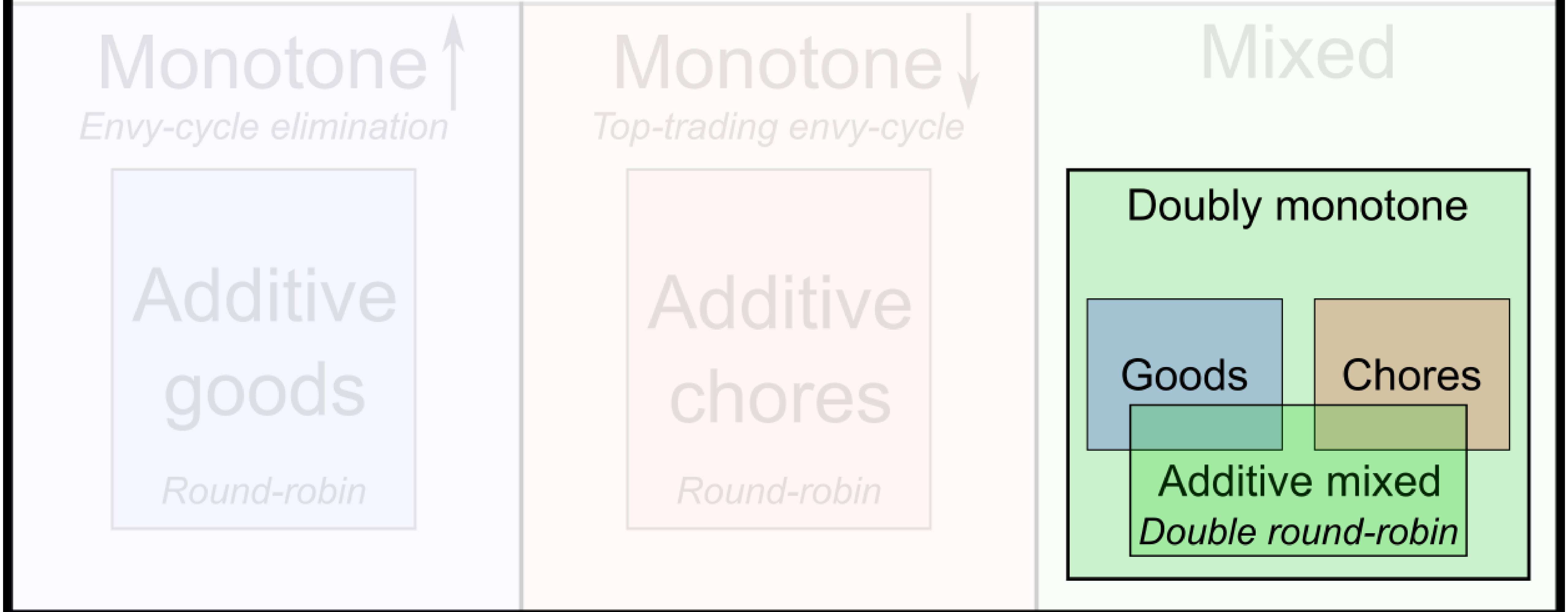
- Assign positive items via envy-cycle elimination
(envy graph defined w.r.t. agents who consider the item a "good")

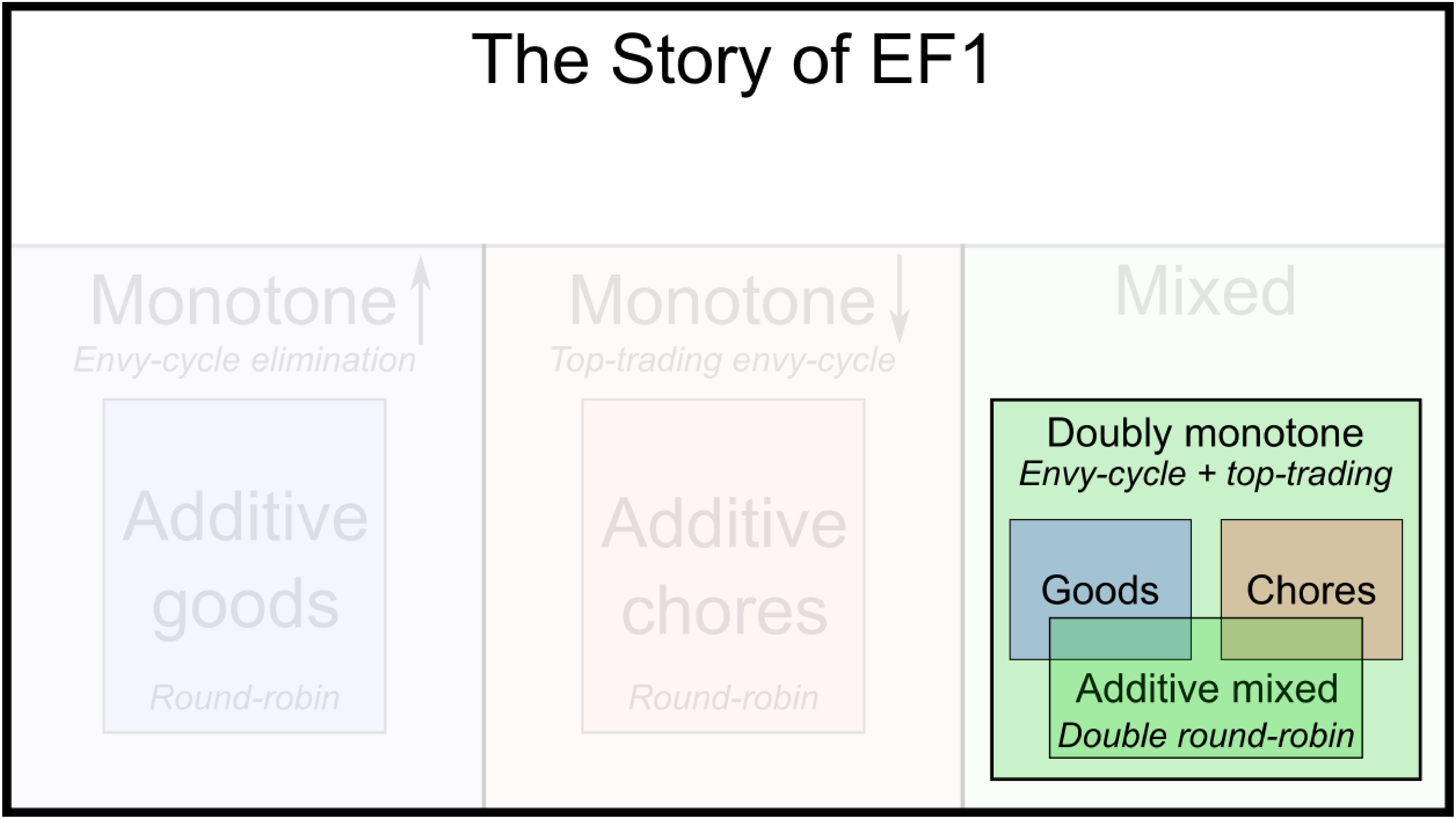
- Assign negative items via top-trading envy-cycle elimination

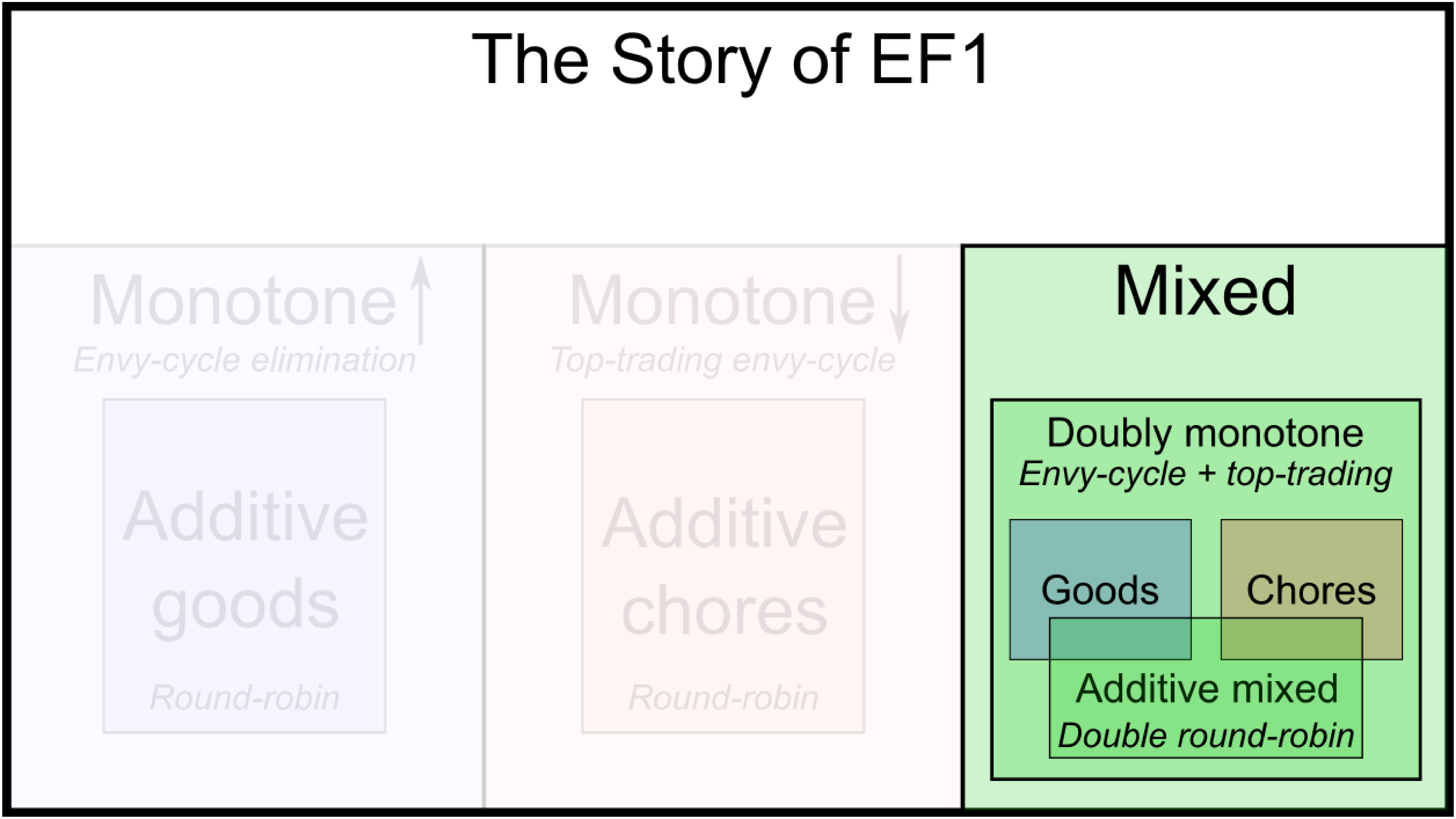
[Bhaskar, Sricharan, and Vaish, *APPROX* 2021]

For doubly monotone items, the above algorithm returns an EF1 allocation.

The Story of EF1




The Story of EF1
Monotone↑
Envy-cycle elimination
Additive goods
Round-robin
Monotone↓
Top-trading envy-cycle
Additive chores
Round-robin
Mixed
Doubly monotone
Envy-cycle + top-trading
Goods
Chores
Additive mixed
Double round-robin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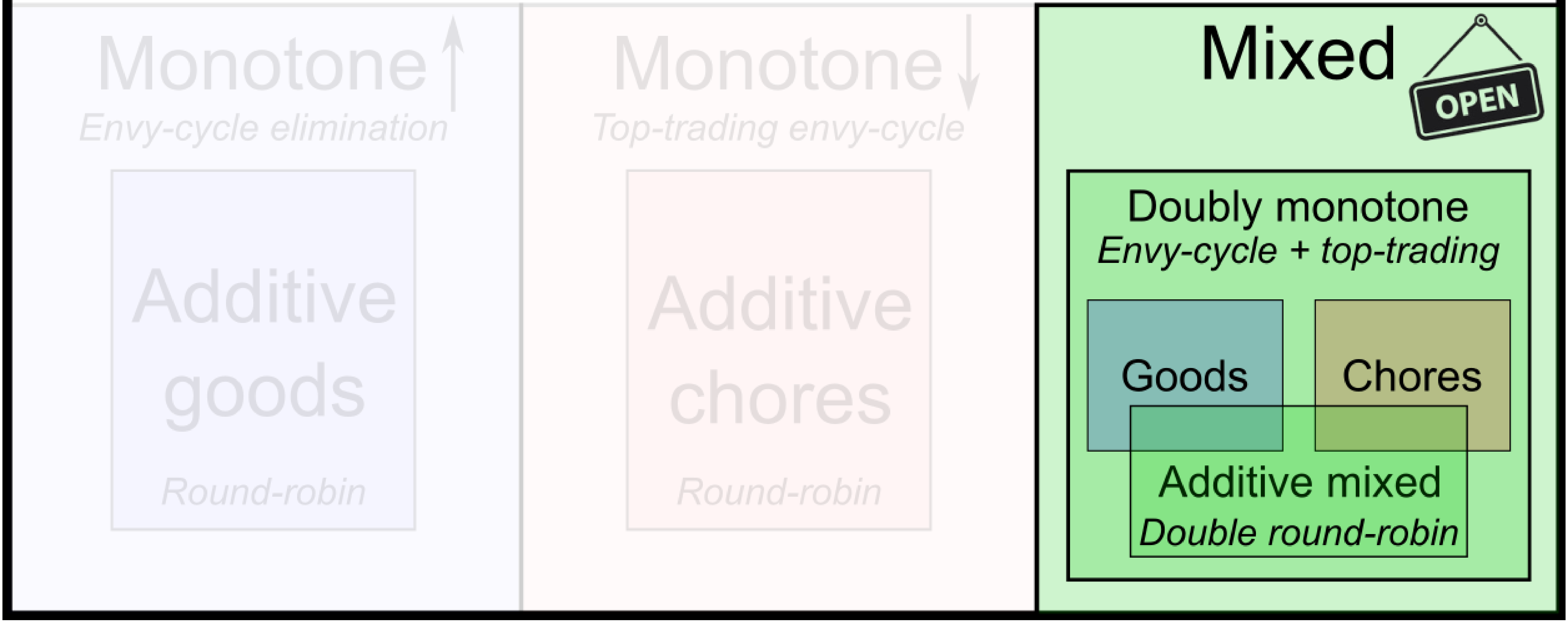
The Story of EF1



Next Time

Fairness via Randomness

Quiz

Quiz

Prove or disprove:

For n identical agents with additive valuations over mixed items, an EFX allocation always exists.

References

- Double round-robin algorithm

  Haris Aziz, Ioannis Caragiannis, Ayumi Igarashi and Toby Walsh
  "Fair allocation of indivisible goods and chores"
  Autonomous Agents and Multiagent Systems, 36(3), 2022 pg 1-21
  https://link.springer.com/article/10.1007/s10458-021-09532-8

- Top-trading envy-cycle elimination

  Umang Bhaskar, A R Sricharan, and Rohit Vaish
  "*On approximate envy-freeness for indivisible chores and mixed resources*"
  APPROX 2021
  https://drops.dagstuhl.de/opus/volltexte/2021/14694/